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On wave propagation characteristics in fluid saturated porous materials by a nonlocal Biot theory

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ABSTRACT

A nonlocal Biot theory is developed by combing Biot theory and nonlocal elasticity theory for fluid saturated porous material. The nonlocal parameter is introduced as an independent variable for describing wave propagation characteristics in poroelastic material. A physical insight on nonlocal term demonstrates that the nonlocal term is a superposition of two effects, one is inertia force effect generated by fluctuation of porosity and the other is pore size effect inherited from nonlocal constitutive relation. Models for situations of excluding fluid nonlocal effect and including fluid nonlocal effect are proposed. Comparison with experiment confirms that model without fluid nonlocal effect is more reasonable for predicting wave characteristics in saturated porous materials. The negative dispersion is observed theoretically which agrees well with the published experimental data. Both wave velocities and quality factors as functions of frequency and nonlocal parameter are examined in practical cases. A few new physical phenomena such as backward propagation and disappearance of slow wave when exceeding critical frequency and disappearing shear wave in high frequency range, which were not predicted by Biot theory, are demonstrated.

1. Introduction

Porous materials, including water or oil saturated sedimentary rocks studied by seismologists and geophysicists, air filled aluminum foam studied in materials engineering and so on, can be found in a variety of geophysical and engineering applications. Biot [1–6] developed a serial of pioneering work on saturated porous materials. Two kinds of dilatational (fast and slow) wave and one rotational plane harmonic wave were predicted for the first time by his theory. In 1980, Plona [7] reported an important observation of a second bulk compressional wave in water saturated sintered glass spheres by experiment. Soon afterwards, Dutta [8] confirmed that the second bulk compressional wave observed in Plona's experiment was consistent with the "slow" compressional wave predicted by Biot theory. Since then, the wave characteristics of saturated porous materials are intensively investigated in many engineering fields, such as seismology [9–11], civil engineering [12], bioengineering [13] and other engineering fields. Also, there are many other researches on saturated porous

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materials both on theory and experiment. One can mention here the experimental and theoretical work of Johnson et al. [14,15], lossless poroelastic wave propagation theory proposed by Geerits et al. [16], stress wave characteristics in anisotropic fluid-saturated porous media proposed by Liu et al. [17]. However, all these studies were developed based on the classical Biot theory.

Although the classical Biot theory has been successfully applied in several engineering fields, the universal applicability is still suspicious as some inconsistencies have been claimed. For example, in the experiment of Lee et al. [18], the phase velocity exhibited a significant negative dispersion relation for frequency range from 0.3 to 1.0 MHz which is different from the prediction of Biot theory. The Biot theory predicted a non-dispersive phase velocity over frequency range from 0.3 to 1.0 MHz. In fact, the wavelength is assumed to be greatly larger than pore size in Biot theory. However, within high frequency range, this assumption cannot stand anymore. The pore size has significant influence on wave characteristics because of scattering when the wavelength is comparable to the pore size. In addition, when a porous material deforms, the deformation difference between solid phase and fluid phase results in perturbation of porosity [19]. If the deformation is linear-elastic and periodic, an inertia force is generated by the periodic fluctuation of porosity which may be called porosity dynamics. It is not difficult to deduce that this inertia force increases with the increasing frequency. However, the porosity dynamics and pore size effect were not included in Biot theory, thus resulted in a deviation from some experimental observations within high frequency range. An alternative elastic theory of porous material was proposed by Lopatnikov and Cheng [20] starting from a macroscopic Lagrangian formulation of poroelasticity, concerning the porosity dynamics. This theory, which introduced the porosity as an independent variable for describing the behavior of porous materials, is a departure from Biot theory. A number of physical phenomena such as dynamic versus static moduli, added mass and porosity dynamics can be interpreted and also a few of new physical phenomena were predicted by this theory. However, the shear wave velocity predicted by this theory is similar to that predicted by Biot theory. It is to say, according to this theory, the porosity dynamics has no effect on shear wave characteristic which is obviously unreasonable. Furthermore, although this theory was obtained with rigorous derivations, it is non-intuitionistic and mathematically complex on engineering, thus limiting application of this theory onto engineering practices.

Because the parameters needed in Biot model are firmly connected with engineering practices and the inherent simplicity of this model, the Biot theory is so far the most widely applied. In order to include the pore size effect and porosity dynamics into the Biot theory, the classical linear-elastic constitutive relation used in Biot theory is replaced by a nonlocal elastic constitutive relation [21] for investigating wave characteristics in the present paper. Chakraborty [22] also proposed a nonlocal poroelastic theory for predicting the negative dispersion wave characteristics in cancellous bone by using the nonlocal theory and Biot theory. However, the model only involved one- and two-dimensional space and some perverse wave propagation characteristics occurred because of the introduction of fluid nonlocal effect. In addition, this nonlocal model that was obtained based on the low-frequency Biot theory is unable to predict the wave characteristics in high frequency range. The present paper is devoted to develop a nonlocal Biot theory by combining nonlocal elasticity [21] and Biot theory [4] from a more general point over full frequency range to study the influences of pore size and fluctuation of porosity on wave characteristics. The wave velocities for both including and excluding fluid nonlocal effect are proposed. Comparison and analysis of results from these two situations confirm that results for excluding fluid nonlocal effect are more reasonable. Similar to Biot theory, two dilatational and one shear waves are predicted by current theory. The wave velocities for these three waves are obtained and the corresponding wave characteristics are analyzed. Good agreements by comparing with the published experimental results are presented, hence verifying the current theory. Wave velocities and quality factors as function of frequency as well as nonlocal parameter are examined utilizing practical cases. By comparing our model with porosity dynamics theory developed by Lopatnikov and Cheng [20], good agreements are achieved for predictions of fast and slow wave velocities. However, according to the prediction of Lopatnikov and Cheng [20], the shear wave velocity is similar to that of Biot theory, which has been deviated from the experimental observation of Bouzidi and Schmitt [23]. From our model, a negative dispersion of shear wave velocity is predicted which agrees well with the experimental results. A few other new physical phenomena such as negative dispersion of wave velocities versus frequency, backward propagation of slow wave, disappearing slow wave and shear wave, are also discussed and demonstrated.

2. Governing equations and solutions

According to the nonlocal elasticity [21], the strain field in the whole body has contributed to the stress at a certain point assumed as **r**. If the body force is neglected, the basic equations for linear, homogeneous, isotropic, nonlocal elastic solid are given by [21]

$$\sigma_{ij,j}=0$$
$$\sigma_{ij}(\mathbf{r})=\int_V \alpha(|\mathbf{r}-\mathbf{r}'|,\tau)C_{ijkl}\varepsilon_{kl}dV(\mathbf{r}')$$
$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{1}$$