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Statistical Energy Analysis with fuzzy parameters to handle populations of structures

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ABSTRACT

Statistical modal Energy distribution Analysis (SmEdA) approach was developed to enlarge the application field of Statistical Energy Analysis (SEA) when equipartition of modal energies is not achieved. SmEdA gives more precise results than standard SEA when compared to exact energy response of a deterministic system in the case of low modal overlap, heterogeneous systems or point excitation. The present paper was initiated by this question: when considering a population of similar structures, each of them being described by SmEdA, do the ensemble averaged energies of subsystem and injected power tend to satisfy SEA equations? In other terms, despite the non-equipartition of energy observed on each element of the population of structures does the ensemble averaging leads to SEA equation where equipartition of energy is assumed? The response to that question that rises from this paper is yes, if the terms of the SEA equation are fuzzy numbers. It results that the energy response given by the model can be interpreted using fuzzy numbers theory.

1. Introduction

In many practical situations, the vibro-acoustic response of complex structures (i.e. automotive, aircraft, submarine, etc.) can be sensitive to small variations introduced by the manufacturing process specially in the mid and high frequency domains [1]. It is then of interest to develop vibro-acoustic models for estimating statistical characteristics of responses of an ensemble of similar structures defined by random parameters. These statistics can be the mean value, the variance, and eventually higher order statistics like Skewness or Kurtosis [2] and the ensemble of structures can be, for instance, the end-products of a production line. The models for describing the uncertainties of this ensemble are generally classified into two categories [3]:

(a) The parametric models which consist in identifying some uncertain physical parameters (i.e. geometrical dimensions, Young modulus, thickness variation, etc.) and in defining models of uncertainty for these parameters. The vibro-acoustic model then propagates the uncertainties through the dynamic behavior of the system to give the statistics of the response. A primary approach can be the Monte Carlo technique which can however be time consuming. Alternative approaches like the fuzzy or interval finite element procedures [4,5] have also been developed.

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(b) The non-parametric models do not introduce uncertainties in physical parameters but use universal model, independent of the origin of the uncertainties. They are more appropriate for the high frequencies where the number of modes of the system can be significant and their properties (i.e. natural frequencies, mode shapes) can be described by these universal models. In the past, Poisson's natural frequency statistics [6] were considered but they are known now to be valid only for symmetric academic systems like rectangular plates or parallelepiped cavities. The Gaussian Orthogonal Ensemble (GOE) [7–9] resulting of the random matrix theory has given more accurate results for more complex cases. The non-parametric models of uncertainty are integrated in dedicated vibro-acoustic models. It is the case for the Statistical Energy Analysis (SEA) [10–12] which is the subject of the present paper. Basically, in SEA, the built-up structure is subdivided into subsystems and the vibration response within each subsystem is characterized by the subsystem energy. For a random broadband excitation in a given frequency band, the energy transfers between the different subsystems are described by the SEA equations. The developments are based on a relation established for two coupled oscillators p_i, p_j excited by uncorrelated white noise forces:

$$\pi_{p_ip_j} = \beta_{p_ip_j}(e_{p_i} - e_{p_j}). \tag{1}$$

It indicates that the power flow exchanged by the two oscillators $\pi_{p_ip_j}$ is proportional to the difference of their total energies, $e_{p_i} - e_{p_j}$. The proportionality coefficient $\beta_{p_ip_j}$ is called the coupling coefficient. The SEA equations expressing the energy exchanged between multi-modal continuous subsystems have been derived from this basic relation by using simplifying assumptions. In particular, it is assumed that the natural frequencies are uniformly distributed in the frequency band of excitation that constitutes the model of uncertainty in SEA. Although this assumption is strong and can be seen as unrealistic from a practical point of view, it is very useful to justify that the power flow relation established for two coupled oscillators excited by uncorrelated white noise forces can be applied to evaluate the power flow between two coupled modes. The ensemble average of the energy sharing between two modes of two different subsystems are deduced with the supplementary assumption that the difference of energies, $e_{p_i} - e_{p_j}$, and the coupling coefficient, $\beta_{p_ip_j}$, are statistically independent,

$$\langle \pi_{p_ip_j} \rangle = \langle \beta_{p_ip_j}(e_{p_i} - e_{p_j}) \rangle \approx \langle \beta_{p_ip_j} \rangle (\langle e_{p_i} \rangle - \langle e_{p_j} \rangle), \tag{2}$$

where brackets indicate an ensemble average on the population of structures considered in SEA.

This supplementary assumption has been clearly highlighted and discussed by Mace in a recent paper [13]. In general, it is not respected because $e_{p_i} - e_{p_j}$ and $\beta_{p_ip_j}$ are strongly correlated. As a consequence, Eq. (2) should be seen as an approximation.

On another hand, SEA supposes that modal energies into a subsystem are uniformly distributed that is not true in general. Studies of Yap and Woodhouse [14], Fredo [15], Finnveden [16], Mace et al. [17,18], Ming and Pan [19], Langley et al. [23] and Lafont et al. [20] illustrate the influence of non-uniformly distributed modal energies on SEA results. When this assumption is not fulfilled, in particular for subsystems with low modal overlap, modern SEA [23] claims to estimate the ensemble mean and variance energy responses of a population of subsystems. Low modal overlap means that the predicted variance becomes large, so that one would not expect the SEA mean to agree very closely with an individual member of the ensemble. This was the key point of our interest in developing SmEdA (Statistical modal Energy distribution Analysis) [25,26] by writing the coupling of subsytems with the Dual Modal Formulation and suppressing the SEA assumption of equipartition of energy. This approach is based on the knowledge of the modal bases of the uncoupled subsystems. By this fact, it is obviously much more time consuming than SEA. However, it proposes a framework to compute explicitly the modal coupling loss factors taking into account geometrical or material complexity of the model. SmEdA, already applied to various industrial structures (car, truck cabin, oil rig, ship), has been developed to better predict energy transmissions of an individual member of the ensemble, in particular in low modal overlap cases that are most often encountered in mechanical structures. However, the consideration of one single deterministic system does not match well with the early developments of SEA which suppose to represent the behavior of the ensemble average of a population of similar systems. This paper is then initiated by the following question: if a nominal system is well predicted by SmEdA and not by SEA, because equipartition of energy is not achieved, does the effect of ensemble average over a population of structures allows the use of SEA equation (i.e. Eq. (2)) to predict the ensemble average energies of subsystems? This study can be put in relation with previously published works: starting from the Energy Influence Coefficients [21,22], Langley and Cotoni [23] derived expressions of the variance of energy for a population of structures. These expressions depend on terms of the standard SEA parameters and additional parameters that describe the variances of the power input and of the coupling between two subsystems. On another hand, Ji and Mace [24] considered two sets of oscillators coupled by springs, they observed that the variance of the excited subsystem depends mainly on the variance of the input power whereas the variance of the receiving subsystem depends on the variance of the intermodal coupling coefficients. The behaviors observed in these two papers [23,24] will be related in the present developments to the modal energy fluctuations in regard to the mean modal energies.

In the present paper, the Dual Modal Formulation and the SmEdA approach are first reminded. Then the ensemble average of a population of structures is studied to see under what conditions SEA equations can be used for ensemble average subsystem energies prediction. It will be shown that SEA equation can be used but Coupling Loss Factors (CLF) and