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Nonlinear free vibrations of centrifugally stiffened uniform beams at high angular velocity

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ABSTRACT

In this paper, we study the bending nonlinear free vibrations of a centrifugally stiffened beam with uniform cross-section and constant angular velocity. The nonlinear intrinsic equations of motion used here are geometrically exact and specific to beams exhibiting large amplitude displacements and rotations associated with small strains. Based on the Timoshenko beam model, these equations are derived from Hamilton's principle, in which the warping is considered. All coupling terms are considered including Coriolis terms. The studied beams are isotropic with clamped-free boundary conditions. By combining the Galerkin method with the harmonic balance method, the equations of motion are converted into a quadratic function treated with a continuation method: the Asymptotic Numerical Method, where the generalized displacement vector is presented as a series expansion. While analysing the effect of the angular velocity, we determine the amplitude versus frequency variations which are plotted as backbone curves. Considering the first lagging and flapping modes, the changes in beam behaviour from hardening to softening are investigated and identified as a function of the angular velocity and the effect of shear. Particular attention is paid to high angular velocities for both Euler–Bernoulli and Timoshenko beams and the natural frequencies so obtained are compared with the results available in the literature.

1. Introduction

The vibrations of rotating beams have attracted the intention of much research because of their interesting applications of the rotary beams in engineering, such as turbine blades, robot arms, wind turbines and helicopter rotor blades. The determination of their dynamic characteristics (natural frequencies and mode shapes) are of great importance in design and control. A significant number of studies have been published on the bending vibrations of rotating beams, with the main objective of predicting the natural frequencies and associated mode shapes, as well as investigating their variations with angular velocity and other effects such as hub radius, taper and shear deformation [1–18]. A review of several studies on rotating beams has been presented by Bazoun [19]. Most of these studies were based on simplified assumptions to arrive at linear and resolvable eigenvalues/eigenvectors problems even for Euler–Bernoulli or Timoshenko beam models. These simplifications are carried out essentially by only taking account of the bending deformations and ignoring the coupling

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Nomenclature

A	cross-sectional area in the undeformed state
$\mathbf{A}_{ki}, \overline{\mathbf{A}}$	matrices used after Galerkin and harmonic balance approximations
$\mathbf{e}_1$	$[1\ 0\ 0]^T$
$\mathbf{F}$	cross-sectional resultant internal force vector
h	hub radius
$\mathbf{H}$	cross-sectional angular momentum vector
$\mathbf{k}$	initial curvature vector
$\mathbf{K}$	deformed beam curvature vector
L	length of beam
$\mathbf{m}$	distributed applied moment per unit length vector
$\mathbf{M}$	cross-sectional internal moment vector
$\mathbf{P}$	cross-sectional linear momentum vector
$\mathbf{V}$	velocity vector of the reference line
x, y, z	cross-sectional reference frame of the undeformed beam
$\overline{y}, \overline{z}$	offsets from the reference line of the cross-sectional mass centre in $(\mathbf{b}_i)$
β	slenderness ratio
δ_h	hub radius ratio
γ	$[\gamma_{11}\ 2\gamma_{12}\ 2\gamma_{13}]^T$
$\kappa_1, \kappa_2, \kappa_3$	twist and bending curvatures $(\kappa_i = K_i - k_i)$
μ	mass per unit length
$\omega(\omega^\star)$	frequency (non-dimensional frequency)
$\Omega(\Omega^\star)$	angular velocity vector (Non-dimensional angular velocity)
$(\bullet'), (\dot{\bullet}), (\tilde{\bullet})$	$\partial(\bullet)/\partial x_1,\ \partial(\bullet)/\partial t,\ -e_{ijk}(\bullet)_k$

terms. Various techniques have been used to approximate the solution of the governing equations, such as the Galerkin [5], Rayleigh–Ritz [1,4], the finite element method (FEM) [3,6,7], the power series [2] and the dynamic stiffness method (DSM) [8]. Apart from these methods for solving the forward problem, an inverse problem approach has been used to study the free vibration of rotating Euler–Bernoulli and Timoshenko beams [20,21]. In such an approach, a polynomial mode shape function is assumed that satisfies the boundary conditions; this function is then used to determine the mass and stiffness variations of the beam. Closed-form solutions are obtained of the governing differential equation.

Rotating beams are subjected to axial deformations induced by centrifugal forces. At high rotating speed, instability occurs at a certain critical value. It was explained in [18,22] that such static instability is explained by the fact that small-deflection (linear) theory is used where large axial deformations induced by centrifugal force take place. The critical angular velocity is characterized by a single bifurcation point where the natural frequency becomes zero. Conventionally, angular velocity is classified as high if the maximum axial steady strain is close to the yield strain [12,15]. For a rotating uniform beam, the maximum steady strain occurs at the root and may be expressed as $\gamma_{11\ \max} = \lambda^2(\delta_h + 0.5)$ [9,15], with $\lambda = \Omega L\sqrt{\rho/E}$ and $\delta_h = h/L$, where Ω, L, h, ρ, E are the angular velocity of the hub, the length of the beam, hub radius, density and Young's modulus, respectively. Most of the studies in the literature deal with low angular velocity (dimensionless angular velocity < 12). However, only a few authors consider high speed rotating beams [11,12,15,23], with the aim of finding a new basis functions and polynomials to ensure a quickly convergence of higher modes at high angular velocity, because it is not evident with usual techniques used to predict these modes at low angular velocity.

Recent studies have focused on the nonlinear vibrations of rotating beams, and most of these approaches are based on different nonlinear dynamic models to construct nonlinear normal modes (NNMs). In [24], the authors use the Euler–Bernoulli beam model and power series to approximate the geometry and the invariant manifolds to construct the nonlinear normal modes. Ref. [25] shows that the Galerkin-based approach yields a more exact reduction than the asymptotic series used in [24], particularly for large amplitudes. In [26], the beam model is based on the von Karman strain–displacement relationships. The cited authors constructed the NNMs by applying the method of multiple scales (MMS) to the Galerkin-reduced discretized version of the equation of motion. Ref. [27] used a dynamic model based on a geometrically exact approach and applied the MMS to construct the NNM. The nonlinearity depends on the angular velocity [17], and the rotating beam may exhibit hardening or softening behaviour depending on the geometric and kinematic conditions [13]. In [28], an algorithm is developed equivalent to the Lyapunov–Schmidt reduction for the computation of nonlinear free vibration curves from a Hopf bifurcation point.

In the present paper, we study the effect of angular velocity on the behaviour of the backbone curve between softening and hardening of a rotating isotropic beam. Our approach is based on the algorithm mentioned above and detailed in [28], which use the intrinsic formulation for dynamics of moving beams developed by Hodges [29,30]. The equations of motion are derived from the Hamilton principle based on a Timoshenko beam model. This formulation is convenient for both isotropic and anisotropic beams with uniform cross-section subjected to large displacements and rotations while undergoing small strain and does not involve any geometrical approximations. It takes advantage of the one dimensional characteristic of a beam, but does not require any definitions of displacements and rotations and assumes that the maximum degree of nonlinearity is equal to two. This model has been used in a study of nonlinear forced vibrations of rotating anisotropic beams [17]. Different techniques of approximations are used in the literature for numerical computation, including the finite element method (FEM) in [31–33], the finite difference method (FDM) in [34] and the Galerkin method in [17,35]. In this present study, we apply a Galerkin approximation to this model, by choosing the weighting functions that represent the assumed modes themselves. This leads to an energy balance and, consequently, provides a better numerical approximation of the solution of the nonlinear beam equations [35–37]. Using this discretization and the harmonic balance