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Internal resonances and dynamic responses in equivalent mechanical model of partially liquid-filled vessel

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ABSTRACT

The paper treats dynamical responses in an equivalent mechanical model for oscillations of a liquid in partially filled vessel under horizontal harmonic ground excitation. Such excitation may lead to hydraulic impacts. The liquid sloshing mass is modeled by equivalent pendulum, which can impact the vessel walls. Parameters of the equivalent pendulum for well-explored case of cylindrical vessels are used. The hydraulic impacts are modeled by high-power potential function. Conditions for internal resonances are formulated. A non-resonant behavior and dynamic response related to 3:1 internal resonance are explored. When the excitation amplitude exceeds certain critical value, the system exhibits multiple steady state solutions. Quasi-periodic solutions appear in relatively narrow range of parameters. Numerical continuation links between resonant regimes found asymptotically for small excitation amplitude, and high-amplitude responses with intensive impacts.

1. Introduction

Vessels filled with liquid are used in many fields of engineering, including nuclear [1], vehicle [2,3] and aerospace industries [4], for storage of chemicals, gasoline, water, and various hazardous liquids [5]. External excitations may cause well-known dynamical effect of liquid sloshing. This effect can take place in liquid cargo on highways, cruises or in stationary vessels exposed to earthquakes. Dynamic loads related to the liquid sloshing may have direct and rather strong hazardous effect on the vessel stability and robustness.

So far, detailed analytical explorations are limited to small-amplitude sloshing in rectangular and cylindrical vessels. While being most interesting and potentially hazardous, high-amplitude liquid sloshing in cylindrical tanks still lacks complete analytic description. The reason is that the liquid in the tank is continuous system with infinite number of degrees of freedom, and its boundary conditions on the free surface are nonlinear and time-dependent. Nevertheless, assessment of the loads created by the high-amplitude liquid sloshing is crucial for design of the vessel and its supports, as well as for establishing the payload limitations [6]. By this reason, a number of approximate phenomenological models were suggested in order to get at least qualitative insight into this phenomenon.

In one of the most well-known phenomenological models, the sloshing dynamics in a partially-filled liquid tank with total mass M is modeled by a pendulum with mass m, length of l, and rotational coordinate θ with respect to the vessel

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center-line. In this simplified model, three dynamic regimes can take place, as shown in Pilipchuk and Ibrahim [7] and presented in Fig. 1:

(1) The liquid free surface performs small oscillations around its trivial stable equilibrium and remains planar. This regime can be successfully described by a pendulum performing small oscillations.
(2) Relatively large oscillations in which the liquid free surface does not remain planar. This motion is described by a differential equation with weak nonlinearity, and can be treated by perturbation methods [7–9]. In this regime the "equivalent" spherical pendulum is considered to perform moderate oscillations, so that approximation $\sin\theta \approx \theta - \frac{\theta^3}{3!}$ is reasonable, and the nonlinearity can be treated as weak.
(3) The free liquid surface is urged into a strongly nonlinear motion, related to liquid sloshing impacts with the tank walls. This regime can be described with the help of a pendulum, which impacts the tank walls.

High-amplitude sloshing can cause hydraulic jumps. In this case (corresponding to Fig. 1c) major hydraulic impacts can act on the vessel structure walls [10]. Despite obvious practical interest, methods for evaluation of the impact in this case are not well developed, and rely primarily on the data of direct experiments and simplified phenomenological models [11]. Hydraulic jumps and wave collisions with the vessel walls give rise to strongly nonlinear behavior in the system, since collisions with rigid or elastic masses cause rapid velocity changes. Hence, even if the sloshing appears due to simple harmonic excitations, the response may be neither harmonic nor periodic. Authors of paper [12] suggest application of methods developed for analysis of a vibro-impact motion. In this work another approach is applied. We use high order smooth potential function to model the interaction between the free-surface wave and the vessel walls, following Pilipchuk [13,14], Pilipchuk and Ibrahim [7] and Gendelman [15]. Smooth polynomial potential function was used by Pilipchuk and Ibrahim [16] to model small amplitude free liquid oscillations in elevated rigid container. The vibro-impact problem of a pendulum oscillating in a rigid container was treated in previous works of Buzhinskii and Stolbetsov [17] and by Shaw and Shaw [18].

Modeling of free-surface oscillations in rectangular tanks with the help of the equivalent pendulum was suggested by Graham in 1951 [19]. Equivalent moment of inertia of a liquid in cylindrical containers has been estimated numerically by Partom [20,21] and verified experimentally by Werner and Coldwell [22]. Parameters of the equivalent pendulum, which corresponds to the first asymmetric sloshing mode of cylindrical and rectangular tanks, were studied by Dodge [23] and Abramson [11]. The non-linear interaction of the sloshing liquid with elastic tank in conditions of parametric excitation was studied by Ibrahim [24], Ibrahim and Barr [25,26], Ibrahim, Gau and Soundararajan [27], and El-Sayad, Hanna and Ibrahim [28]. Observations and experiments show that in the vicinity of internal resonances violent response might take place. Using

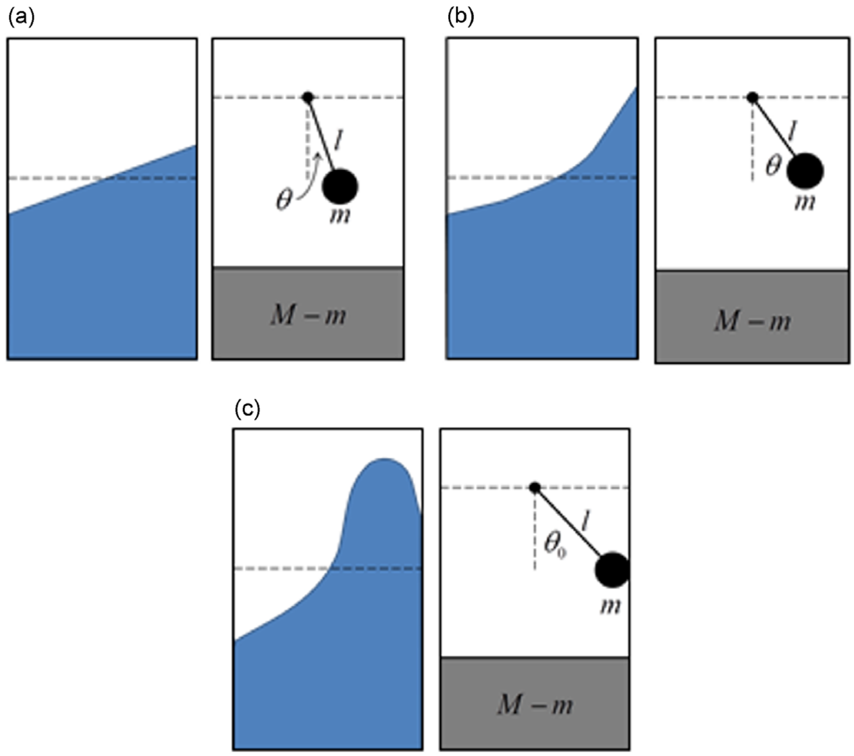

Fig. 1. Regimes of liquid free-surface motion and their equivalent mechanical models.

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