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Tangential acceleration feedback control of friction induced vibration

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ABSTRACT

Tangential control action is studied on a phenomenological mass-on-belt model exhibiting friction-induced self-excited vibration attributed to the low-velocity drooping characteristics of friction which is also known as Stribeck effect. The friction phenomenon is modelled by the exponential model. Linear stability analysis is carried out near the equilibrium point and local stability boundary is delineated in the plane of control parameters. The system is observed to undergo a Hopf bifurcation as the eigenvalues determined from the linear stability analysis are found to cross the imaginary axis transversally from RHS s-plane to LHS s-plane or vice-versa as one varies the control parameters, namely non-dimensional belt velocity and the control gain. A nonlinear stability analysis by the method of Averaging reveals the subcritical nature of the Hopf bifurcation. Thus, a global stability boundary is constructed so that any choice of control parameters from the globally stable region leads to a stable equilibrium. Numerical simulations in a MATLAB SIMULINK model and bifurcation diagrams obtained in AUTO validate these analytically obtained results. Pole crossover design is implemented to optimize the filter parameters with an independent choice of belt velocity and control gain. The efficacy of this optimization (based on numerical results) in the delicate low velocity region is also enclosed.

1. Introduction

The deleterious aspect of friction-induced instability is common to most mechanical systems, e.g. bearings, transmissions, hydraulic and pneumatic cylinders, valves, brakes, lead screws, oil well drill-strings and wheels and nevertheless in control systems like in various drive systems, high-precision servo mechanisms, robots, pneumatic and hydraulic systems, in position control systems and anti-lock brakes for cars, etc. So, the undesirable self-excited vibrations induced by non-linear character of friction in linearly stable systems has motivated various researchers worldwide to investigate into the inherent laws of friction and unanimously put forth various models that can perfectly imitate the frictional phenomena in original systems as well as effective methods to control this undesirable phenomenon that leads to significant inconvenience.

Friction-induced self-excited vibrations are attributed to mainly three mechanisms, namely, the velocity weakening characteristics of friction force which is also known as the Stribeck effect, mode-coupling and sprag-slip instabilities. A detailed review of these basic mechanisms is assembled in [1,2].

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Nomenclature

A_1, A_2	amplitudes of non-linear oscillations of the system and the filter respectively
C	viscous damping coefficient
$c = \frac{C}{M\omega_n}$	non-dimensionalized viscous damping coefficient
F	friction force between mass and belt
J, J_1	Jacobian of the linearized flow around the equilibrium, Jacobian of the non-linear flow around the equilibrium
K	oscillator stiffness
k_c^*	gain of power amplifier
$k_c' = k_c^*/X_0$	non-dimensionalised gain of power amplifier
k_1^*	sensor gain
$k_1 = X_0 k_1^*$	non-dimensionalised sensor gain
$k_c = k_1 k_c'$	non-dimensionalised loop gain
M	oscillator mass
$N0$	normal load
t	time
v_b	belt velocity
$v_0 = \frac{v_b}{\omega_n X_0}$	Non-dimensionalised belt velocity
X	displacement of oscillator mass
X_c	controlled displacement of base
$X_0 = \frac{N_0}{M\omega_n^2}$	reference displacement
x_f	non-dimensional filter variable
$y = \frac{X}{X_0}$	non-dimensional displacement of mass
$z = y - y_0$	coordinate transformation where (y_0, x_{f0}) is the static equilibrium of linearized system
$\alpha^*, \alpha = \alpha^* \omega_n X_0$	model parameter that determines the slope of the friction-velocity curve in the low velocity range, non-dimensional form of the same
$\beta* > 1,\ \beta = \beta^* \omega_n X_0$	a very large positive quantity, non-dimensional form of the same
γ	$\Delta\mu e^{-\alpha v_0}$
$\varepsilon \ll 1$	an infinitesimally small quantity
λ_1, λ_2	identical complex conjugate pair of poles used in pole cross-over design
$\mu, \Delta\mu$	minimum kinetic coefficient of friction, the difference between the static friction coefficient and the minimum kinetic friction coefficient
ξ_f	damping ratio of second-order filter
$\tau = \omega_n t$	non-dimensionalised time
ϕ_1, ϕ_2	phases of non-linear oscillations of the system and the filter respectively
$\tilde{\omega}_f, \omega_f$	natural frequency of the second-order filter, non-dimensional form of the same
$\omega_n = \sqrt{\frac{K}{M}}$	natural frequency of the mechanical system

Literary contributions suggesting various passive and active control methods to eliminate or effectively reduce these unwanted friction-induced vibrations are extensive. Among the passive control methods,high frequency tangential excitation for controlling these self-excited vibrations is proposed by Thomsen [3] and is more elaborately studied by Feeny and Moon [4] through experimentation. Chatterjee [5] also uses this technique on sophisticated microscopic models. Use of dynamic vibration absorbers are investigated by Popp and Rudolph [6] and Chatterjee [7]. In the domain of the active vibration control methods, time-delayed feedback control methods have gained a remarkable popularity. Elmer [8] considers the time-delayed state feedback control of normal load. Das and Mallik [9] use an experimental setup of friction-driven system to propose the efficacy of a time-delayed PD feedback control of forced vibration as well as friction-induced vibrations due to Stribeck effect. Chatterjee [10] presents the time-delayed displacement feedback control of different types of friction-induced instabilities. Chatterjee and Mahata [11] introduces a novel method called ‘recursive time-delayed acceleration feedback control’ where the control force is devised as an infinite weighted sum of the acceleration of the vibrating mass measured at regular intervals. Heckl and Abrahams [12] proposed a method of active control of friction-induced vibration by superimposition of a tangential force on the friction force. Chatterjee [13] presents a novel active control method by normal load modulation depending upon the state of the oscillatory system. Stribeck effect is considered to cause the friction-induced vibrations and Lyapunov's second method is used to derive the basic control law. Chatterjee [14] makes use of the actively controlled impulsive forces generated by expansion and contraction of a PZT (lead zirconium titanate) to control these self-excited oscillations. von Wagner et al. [15] demonstrated an active control technique of suppressing squeal in floating calliper disc brake by the use of “smart pads”.

This paper considers the efficacy of a tangential control in arresting the self-excited vibrations generated due to the velocity-weakening friction force. An archetypal model of a spring-mass-damper system on a moving belt is studied where the acceleration of the mass is fed to a controller which in turn actuates the tangential control through an actuator fitted to the base of the vibratory system. Linear stability analysis and nonlinear analysis of the friction-system is performed by describing the friction phenomena as given by the exponential friction model proposed by Hinrichs et al. [16]. Local and global stability boundaries are constructed in the plane of control parameters. Direct numerical simulations are shown to substantiate the analytically obtained results. Pole crossover optimization is performed to design the controller parameters and their dependence with the system parameters are found henceforth.

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