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The time domain moving Green function of a railway track and its application to wheel–rail interactions

X. Sheng*, X. Xiao, S. Zhang

State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu, Sichuan, China

A R T I C L E I N F O

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A B S T R A C T

When dealing with wheel-rail interactions for a high-speed train using the time domain Green function of a railway track, it would be more reasonable to use the moving Green function associated with a reference frame moving with the train, since observed from this frame wheel/rail forces are stationary. In this paper, the time domain moving Green function of a railway track as an infinitely long periodic structure is defined, derived, discussed and applied. The moving Green function is defined as the Fourier transform, from the load frequency domain to the time domain, of the response of the rail due to a moving harmonic load. The response of the rail due to a moving harmonic load is calculated using the Fourier transform-based method. A relationship is established between the moving Green function and the conventional impulse response function of the track. Properties of the moving Green function are then explored which can largely simplify the calculation of the Green function. And finally, the moving Green function is applied to deal with interactions between wheels and a track with or without rail dampers, allowing non-linearity in wheel-rail contact and demonstrating the effect of the rail dampers.

1. Introduction

Train-track dynamic interactions are responsible for many issues concerning the railway industry, such as roughness on wheel-rail rolling surfaces, rail corrugation, wheel out-of-round and wheel-rail noise. Simulation tools are useful to gain insights into these issues and several packages are commercially available. In case of high-speed train, train-track interactions are of high frequencies, and involve complex vibrational resonances and propagations in the track due to the high train speed. Regular impacts between wheel and rail occur if discrete wheel irregularities are present.

Different approaches to modelling train-track interactions have been established. To get deeper insights, especially for high speed and high frequency, modelling approaches are still being improved, and even innovated.

One of the important tasks is to model the track and predict track dynamics of high frequency. For a modern railway track with sleepers, it is normally idealised to be an infinitely long periodic structure. The period is equal to the sleeper spacing for a conventional ballasted track, or equal to the length of a slab plus the small slab-slab gap if the track is a non-ballasted and pre-cast slab track such as those used for high-speed railways in China. An important aspect in dealing with track dynamics is to calculate the response to moving or stationary harmonic loads. Results from such calculations can

* Corresponding author. Tel.: +86 28 87634209; fax: +86 28 87600868.
E-mail address: shengxiaozhen@hotmail.com (X. Sheng).

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Nomenclature

$\mathbf{q}$	displacement vector of the rail
$\mathbf{Q}$	receptance matrix of the rail
$\hat{\tilde{\mathbf{Q}}}_j$	inverse Fourier transform of $\tilde{\mathbf{Q}}_j$
$\mathbf{G}$	moving Green function matrix
$\hat{\mathbf{f}}(\Omega)$	Fourier transform of $\mathbf{f}(t)$
P_{k0}	half the axle load of the kth wheel
m_k	mass of the kth wheel
$w_k^W(t)$	vertical displacement of the kth wheel
z_k	wheel-rail irregularity at the kth wheel-rail contact point
L	sleeper spacing
$\dot{w}_k^W(t_n)$	velocity of the kth wheel at instant t_n
C_k	constant in the contact model for the kth whee-rail contact
$\mathbf{p}_0$	amplitude vector of harmonic loads applied on the rail
$\tilde{\mathbf{Q}}_j$	jth Fourier coefficient matrix of the receptance matrix of the rail
$\mathbf{G}^*$	stationary Green function matrix
$\mathbf{f}(t)$	moving force vector applied on the rail
$F_k(t)$	the kth vertical wheel-rail force
$f_k(t)$	the kth dynamic wheel-rail force
a_k	initial position of the kth wheel
$w_k^R(t)$	vertical displacement of the rail at the kth wheel-rail contact point
G^w	moving Green function associated with rail vertical displacement
M	number of wheels
$w_{k0}^R(t_n)$	rail displacement at the kth wheel-rail contact point due to the moving axle loads
R_k	radius of the kth wheel

reveal dynamic characteristics (including modal resonances and wave propagations) of the track and at the same time provide a basis, either in the time domain or in the frequency domain, for dealing with train-track interactions.

It is always attractive to deal with high-frequency vibration problems in the frequency domain. In many earthlier wheel–rail interaction models, e.g. [1], the motion of the wheels is replaced by the motion of a roughness strip, resulting in a linear, time-invariant system. Since wheels are forced to be stationary in the track direction, this 'moving-roughness approach' is able to consider the roughness excitation without including the effect of the wheel speed, but totally excludes excitations from moving axle loads. Furthermore, for some roughness wavelengths this approach changes the vibration propagation characteristics of the rail, i.e., it replaces a propagating vibration mode with a non-propagating one, or vice versa [2]. For high-speed train, this approach seems to be inappropriate.

Train-track interactions can be studied in the time domain by solving differential equations of motion of the train-track system. This approach requires the track to be truncated into a finite length. To minimise wave reflections from the two ends, the track model must be sufficiently long. For a high-speed train, the entire train may be considered and the track model must be much longer than the train [3]. This is due to the strong inter-vehicle couplings and long-distance propagating vibration waves in the track induced by the fast-moving train. Normally vibration of the rail is modelled using either the modal superposition method [3] or the finite element method [4]. Both would generate a large number of differential equations of time-varying coefficients. In order to account for high frequency and high train speed, a very small time step is required to solve such a large number of differential equations. Recently, impact noise from wheel flats based on measured profiles is predicted in Ref. [5] using the modelling approach of [4] with a detailed account for the contact mechanics.

The second time domain approach is based on the idea of 'mass on time-varying spring' [6–8]. Here the time-varying spring represents the track which provides varying dynamic stiffness as a wheel rolls over a sleeper bay. In this approach, receptances of the rail are obtained using a model in the frequency domain and rational fraction polynomials are chosen to best fit these receptances. Then the inverse Laplace transform is employed to transform the dynamics of the track into a small set of ordinary differential equations. These differential equations and those for vehicles are coupled via wheel-rail contact conditions to constitute a set of differential equations governing vehicle-track dynamic interactions. It is noticed that in Refs. [7,8], the calculation of rail receptances is carried out in a quasi-static manner.

Another approach is the time domain Green function method [7,9,10], in which integral, in addition to differential, equations are solved. According to Refs [9,10], the time domain Green function, which is the response of the rail at a position due to an impulsive force (a delta-function) applied at the same or another position, is first calculated. Then the classic Duhamel integral equation is applied to express responses of the rail at wheel-rail contact points due to wheel-rail forces, and at the same time differential equations are set up for the wheels. These equations are then coupled using wheel-rail contact conditions. Travelling along the rail of a wheel is accounted for by changing the response and loading positions properly in the Green function. To do so, Green functions have to be calculated for every loading positions within a sleeper bay. Refs. [11,12] have used the time domain Green function method to deal with wheel-rail impact and noise caused by a wheel flat.

It is observed that Green functions used in [9,10] are calculated without the effect of load speed. Therefore these Green functions are 'stationary' ones. It is also noticed that the time-domain Green function in Refs. [11,12] is also calculated in a 'stationary' sense based on the finite element model developed in Ref. [4] of a finite length of track.

It would be more reasonable to use the Green function which is associated with a reference frame moving with the train, since observed from this frame wheel-rail forces are stationary. Such a Green function may be termed the time-domain moving Green function. This idea has already been explored recently in Refs. [13–15]. It should be noted that in these papers,