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Ray and wave scattering in smoothly curved thin shell cylindrical ridges

Niels Søndergaard [a], David J. Chappell [b,*]

[a] inuTech GmbH, Fürther Straße 212, 90429 Nuremberg, Germany
[b] School of Science and Technology, Nottingham Trent University, Nottingham NG11 8NS, United Kingdom

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ABSTRACT

We propose wave and ray approaches for modelling mid- and high-frequency structural vibrations through smoothed joints on thin shell cylindrical ridges. The models both emerge from a simplified classical shell theory setting. The ray model is analysed via an appropriate phase-plane analysis, from which the fixed points can be interpreted in terms of the reflection and transmission properties. The corresponding full wave scattering model is studied using the finite difference method to investigate the scattering properties of an incident plane wave. Through both models we uncover the scattering properties of smoothed joints in the interesting mid-frequency region close to the ring frequency, where there is a qualitative change in the dynamics from anisotropic to simple geodesic propagation.

1. Introduction

Thin shell components can be found in many large built-up mechanical structures such as cars, ships, and aeroplanes. The prediction of the mid- and high- frequency vibrational properties of these structures becomes computationally prohibitive for standard element-based methods, such as the finite element method [1]. The main reasons for this limitation are: firstly, very fine meshes are required for an adequate representation of the highly oscillatory wave solutions and the computational complexity grows with frequency raised to the power of the dimension of the space being modelled. Secondly, small uncertainties originating from the manufacturing process lead to a much larger variability in vibro-acoustic responses in the high frequency range [2], meaning that the response of any individual manufactured structure is of less interest to computer aided engineering practitioners than the average responses.

Methods such as statistical energy analysis (SEA) [2] and ray tracing [3] are more commonly applied for modelling wave problems at high frequencies. SEA has traditionally proved more popular for structural vibration problems with low damping [4], whereas ray tracing has found its niche in applications where relatively few reflections need to be tracked in computer graphics [3], room acoustics [5] and seismology [6]. Ray tracing has also been applied to elastic wave transmission problems on shells and plates [7]. In this context the ring frequency, that is the frequency above which longitudinal waves in a curved shell behave as they would in a flat plate, provides a useful point of reference. Beyond the ring frequency the ray dynamics in a curved shell are relatively simple following geodesic paths, but below the ring frequency one finds that asymptotic ray theories show richer features. The dispersion relations become highly anisotropic with likewise anisotropic

* Corresponding author.

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propagation [8]. An SEA treatment would fail to capture the non-trivial way in which the curved shell geometry influences the wave and ray propagation below the ring frequency [9], and hence ray methods can provide useful insight [7].

Wave scattering from discontinuous line joints below the ring frequency has been considered in [9,10]. However, manufacturers of large built-up mechanical structures are increasingly developing larger and lighter sub-components, whereby large thin shell structures are replacing more traditional plate-beam and multi-plate assemblies. The manufacturing process for such thin shells often entails casting molten metal (for example aluminium), which gives rise to curved components with smooth transitions between flat and curved regions. This raises the question of how the ray and wave scattering, and hence the vibrational properties of the structure, are modified in these smooth designs. In this work we study the case of a singly curved shell, chosen here as an assembly of two plates joined smoothly with a quarter of a cylinder. This simplified assembly represents a typical curved region within one of the larger sub-structures described above. We shall go beyond plane waves and ray tracing calculations by solving the full wave scattering problem numerically.

The numerical solution to the full wave scattering problem will be discussed in comparison with the corresponding ray tracing calculations. In both cases we find effective laws for the scattering properties, which may be inserted into ray or wave propagation modelling techniques such as dynamical energy analysis (DEA) [11,12] or the wave and finite element method [13,14]. In particular, combining these local scattering models within a larger model of a built-up structure will lead to a hybrid method for structures including curved thin shell components in the mid-frequency regime. Here, the natural definition of the mid-frequency regime is given by the range of frequencies that are high enough for a pure FEM analysis to be impractical, but low enough so that a simple geodesic description of the trajectory evolution is invalid. In the high-frequency case, DEA [11] can be applied to model the vibrational energy transport of a built-up structure along geodesic paths using the mesh data from a FEM analysis [12]. In fact, DEA presents a link between ray tracing and SEA by casting the wave or ray problem into the language of evolution operators. We note that an equivalent operator formalism has also been long known in computer graphics [15], although the theory underlying DEA arose from the more general setting of evolution operators for transporting flows in dynamical systems [16,17].

The organisation of the article is as follows: we introduce the necessary shell theory and derive a wave scattering model for a singly curved shell in Section 2. We then present two approaches for solving the wave scattering model; a short wavelength asymptotic ray tracing model based on this shell theory is detailed in Section 3, and a finite difference discretisation of the full wave model is described in Section 4. We then discuss and compare numerical results for both the wave and ray scattering models, and the resulting reflection and transmission laws in Section 5.

2. Thin shell wave theory

2.1. Governing equations of Donnell's shell theory

The thin shell theory of Donnell is one of the simplest and widely adopted models [7,18]. In this theory, moments and transverse forces are expressed by the displacement w of the middle surface as known from the theory of laterally loaded plates. As with other theories of continuum mechanics, shell theory is formulated in tensor form [19]. Some properties of tensors are summarised in Appendix A. We assume an isotropic shell of thickness h, Young's modulus E, density ρ and with Poisson ratio ν. The displacement vector of a point originally on the mid-surface of the shell is decomposed into tangential and normal components thus $\mathbf{u}=[u^1\ u^2\ w]^{\mathrm{T}}$.

The following tensor equation for the normal displacement w may be derived [7]

$$\rho h\frac{\partial^2 w}{\partial t^2}=-D_\alpha D_\beta\left(B(1-\nu)D^\alpha D^\beta w\right)-D_\alpha D^\alpha\left(B\nu D_\beta D^\beta w\right) \\ -C((1-\nu)d^\alpha_\beta\epsilon^\beta_\alpha+\nu d^\alpha_\alpha\epsilon^\beta_\beta), \tag{1}$$

where

$$B=\frac{Eh^3}{12(1-\nu^2)} \quad \text{and} \quad C=\frac{Eh}{1-\nu^2} \tag{2}$$

are the bending and extensional stiffness, respectively. All Greek alphabet indices take values from the set $\{1,\ 2\}$. Also, the membrane strain is

$$\epsilon_{\alpha\beta}=\frac{1}{2}(D_\alpha u_\beta+D_\beta u_\alpha)+d_{\alpha\beta}w. \tag{3}$$

with $d_{\alpha\beta}$ the second fundamental form and D^α the covariant derivative. These are discussed further in the next section, where they are simplified for a singly curved shell. The tangential displacements $(u^1,\ u^2)$ in the directions (x^1,x^2), respectively, satisfy [7]

$$\rho h\frac{\partial^2 u^\alpha}{\partial t^2}=D_\beta\left(C\left((1-\nu)\epsilon^{\alpha\beta}+\nu\epsilon^\gamma_\gamma g^{\alpha\beta}\right)\right). \tag{4}$$

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