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Stochastic averaging based on generalized harmonic functions for energy harvesting systems

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ABSTRACT

A stochastic averaging method is proposed for nonlinear vibration energy harvesters subject to Gaussian white noise excitation. The generalized harmonic transformation scheme is applied to decouple the electromechanical equations, and then obtained an equivalent nonlinear system which is uncoupled to an electric circuit. The frequency function is given through the equivalent potential energy which is independent of the total energy. The stochastic averaging method is developed by using the generalized harmonic functions. The averaged Itô equations are derived via the proposed procedure, and the Fokker-Planck-Kolmogorov (FPK) equations of the decoupled system are established. The exact stationary solution of the averaged FPK equation is used to determine the probability densities of the amplitude and the power of the stationary response. The procedure is applied to three different type Duffing vibration energy harvesters under Gaussian white excitations. The effects of the system parameters on the mean-square voltage and the output power are examined. It is demonstrated that quadratic nonlinearity only and quadratic combined with properly cubic nonlinearities can increase the mean-square voltage and the output power, respectively. The approximate analytical outcomes are qualitatively and quantitatively supported by the Monte Carlo simulations.

1. Introduction

Energy harvesting from mechanical energy to support low-consumed electronics has arose as an outstanding research field and continues to grow rapidly. There are several prominent and comprehensive review papers and monographs, especially Tang et al. [1], Pellegrini et al. [2], Harne and Wang [3], Daqaq et al. [4], Erturk and Inman [5] and Elvin and Erturk [6], introducing the development and the situation of energy harvesting.

To enhance the broadband feature of vibration-based energy harvesters (VEHs), researchers have proposed several prominent strategies, such as linear resonance frequency tuning [7], multimodal VEHs [8], and stiffness nonlinearities characteristics [3,4]. Mann and Sims [9] designed a novel energy harvesting mechanism which uses a magnet to generate a nonlinear restoring force, employed the method of multiple scales to determine the response, and validated the response via the experimental data. Erturk and Inman [10] constructed a piezomagnetoelastic energy harvester and obtained numerically

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and experimentally the voltage response under harmonic excitation. Zhu and Zu [11] designed a buckled-beam piezoelectric energy harvester to improve the bandwidth of the output voltage by introducing a midpoint magnetic force. Zhou et al. [12] presented numerical and experimental data of a bistable piezomagnetoelastic energy harvester with rotatable magnets that improved broadband frequency voltage response.

Since randomness inherent in most real-world environments may essentially change the characteristic of energy harvesters, many researchers treated vibration-based energy harvesting under random excitations with all sorts of stochastic approaches. Cottone et al. [13] found the bistable energy harvester is superior to the linear ones under stochastic excitation via numerical simulation and experiment. However, Daqaq [14] illustrated that monostable Duffing oscillator cannot afford any enhanced power than the relevant linear system under Gaussian white noise and colored noise excitations. Daqaq [15] derived an approximate expression for the mean power of vibration energy harvester subject to exponentially correlated noise and proved that there is an optimal potential shape leading to maximize the output power. Green et al. [16] reported that Duffing-type nonlinearities of electromagnetic energy harvesting can reduce their output power and verified the tendency using the technique of equivalent linearization. Ali et al. [17] established a closed-form approximate power expression of bistable piezoelectric energy harvester under random excitation, and validated the analytical predict values against the Monte Carlo simulation results. Masana and Daqaq [18] calculated the voltage response of buckling piezoelectric beam under band-limited noise, discussed the influence of stiffness-type nonlinearities on the displacement and voltage response. Daqaq [19] presented the voltage response statistics by introducing the method of moment differential equations of FPK equation and demonstrated that the time constant ratio of the energy harvester plays a key role in developing the performance of nonlinear harvesters under the random environment. He and Daqaq [20,21] employed the statistical linearization techniques and finite element method of the FPK equation to investigate the effects of the potential energy function on the mean steady-state approximate output power. Xu et al. [22] proposed a novel decoupling technique to develop a stochastic averaging of energy envelope for Duffing-type vibration-based energy harvesters, and discussed the effects of the system parameters on the mean square output voltage and power. Kumar et al. [23] used the finite element method to solve the FPK equation of the associated bistable energy harvester, and analyzed the effects of the system parameters on the mean square output voltage and power. Jin et al. [24] introduced the generalized harmonic transformation to decouple the electromechanical equations, and applied the equivalent nonlinearization technique to derive a semi-analytical solution of the corresponding nonlinear vibration energy harvesters subjected to Gaussian white noise excitation.

The Fokker–Planck–Kolmogorov equation provides a powerful tool for treating the statistical characteristics of nonlinear stochastic system. The solutions of Fokker–Planck–Kolmogorov equation define the time-dependent evolutions of probability densities. However, exact solutions are usually difficult to obtain for nonlinear stochastic systems. In fact, they have been found only in a few special cases. The stationary solution, namely, the steady-state probability density, is relatively obtainable and useful in some practical circumstances. The FPK equation of the coupled electromechanical system is a three-dimensional nonlinear partial differential equation. Its exact solution is difficult to obtain, even for exact stationary probability densities. To date all known exact stationary solutions of the FPK equation have been obtained only for degraded cases of the decoupled electromechanical system [14–16]. Therefore some approximate methods for solving the FPK equation of the coupled electromechanical system have been reported which included the statistical linearization techniques [17,20,21], the moment differential equations method [19], the finite element method [20,21,23], the stochastic averaging of energy envelope [22], the equivalent nonlinearization technique [24].

Among various approaches to nonlinear random vibration, the stochastic averaging method is a powerful approximate technique for the prediction of response of nonlinear system subject to external and parametric random excitations. The success of the stochastic averaging method is mainly due to the reduction of dimensions of the FPK equation while the essential behavior of the system is retained. It is also a convenient approximate approach to predict the stationary response of nonlinear stochastic systems and has been extensively used in theory and engineering application of random vibration. There are several excellent and comprehensive review papers, observably Roberts [25], Crandall and Zhu [26] and Zhu [27], reviewing the stochastic averaging methods in different times. Roberts [28] employed the stochastic averaging method to investigate the response of ship rolling motion and obtained the exact stationary probability density function. Roberts and Spanos [29] applied the stochastic averaging method to study the response of nonlinear oscillator under external excitation with or without combined parametric excitation, and obtained the exact stationary and non-stationary probability density function. Huang et al. used the generalized harmonic functions averaging method to predict the response of Duffing-van der Pol oscillator under combined harmonic and white-noise excitations [30] and wide-band random excitation [31]. So far, to the authors' best knowledge, there is no the generalized harmonic functions stochastic averaging analysis on energy harvesting. To address the lacks of research in this aspect, the present work develops the stochastic averaging technique to determine the response of nonlinear energy harvesters under Gaussian white noise excitation. The developed approach can quantitatively account for the effects of nonlinearities on the mean-square voltage and the output power. It may also provide a foundation for further qualitative investigations such as bifurcation analysis, while bifurcations are not treated.

The paper is organized as follows. Section 2 reviews the generalized harmonic functions of a nonlinear conservative oscillator. Section 3 introduces the basic model of nonlinear vibration energy harvesters. Section 4 decouples the electric circuit system and the mechanical system of nonlinear vibration energy harvesters. Section 5 presents the stochastic averaging procedure of nonlinear vibration energy harvesters by using the generalized harmonic functions. Sections 6–8