Journal of Sound and Vibration 376 (2016) 95–111

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Topology optimization of two-dimensional elastic wave barriers

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ARTICLE INFO

Article history:
Received 9 September 2015
Received in revised form
29 February 2016
Accepted 28 April 2016
Handling Editor: I. Lopez Arteaga
Available online 16 May 2016

Keywords:
Topology optimization
Elastodynamic wave propagation
Vibration reduction measures
Wave barriers

ABSTRACT

Topology optimization is a method that optimally distributes material in a given design domain. In this paper, topology optimization is used to design two-dimensional wave barriers embedded in an elastic halfspace. First, harmonic vibration sources are considered, and stiffened material is inserted into a design domain situated between the source and the receiver to minimize wave transmission. At low frequencies, the stiffened material reflects and guides waves away from the surface. At high frequencies, destructive interference is obtained that leads to high values of the insertion loss. To handle harmonic sources at a frequency in a given range, a uniform reduction of the response over a frequency range is pursued. The minimal insertion loss over the frequency range of interest is maximized. The resulting design contains features at depth leading to a reduction of the insertion loss at the lowest frequencies and features close to the surface leading to a reduction at the highest frequencies. For broadband sources, the average insertion loss in a frequency range is optimized. This leads to designs that especially reduce the response at high frequencies. The designs optimized for the frequency averaged insertion loss are found to be sensitive to geometric imperfections. In order to obtain a robust design, a worst case approach is followed.

1. Introduction

This paper focuses on reducing two-dimensional wave transmission in an elastic halfspace by designing a wave barrier using topology optimization. Topology optimization searches for the most efficient distribution of a given amount of material in a specified design domain [1]. The problem is formulated as a mathematical optimization problem where the performance of the structure is maximized while satisfying a set of constraints such as the volume fraction of the stiffened material. Topology optimization simultaneously optimizes not only the size and the shape of the design, but also the topology, making it possible to obtain novel, improved design geometries.

Topology optimization was originally developed for static mechanical problems, but has since then been used for a variety of applications including problems governed by wave propagation [2]. A lot of research has been performed in the

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http://dx.doi.org/10.1016/j.jsv.2016.04.039
0022-460X/

field of photonic crystal waveguide design. These electromagnetic waveguides are designed based on the band-gap phenomenon in periodic structures, obstructing wave propagation for specific frequencies [3,4].

Next to photonic band-gaps due to electromagnetic waves, also phononic band-gaps due to elastic and acoustic waves have been investigated. Sigmund and Jensen [5] maximize the bandwidth of phononic band-gap materials and minimize the transmitted wave amplitude of a band-gap structure subjected to harmonic loading. For a one-dimensional homogeneous elastodynamic medium, topology optimization leads to periodic structures where the thicknesses of the layers are equal to a quarter of the considered wavelength [6]. Jensen [7] discusses a two-dimensional problem, where an incoming plane pressure or shear wave is maximally reflected or dissipated by an optimal periodic distribution of scattering or absorbing inclusions.

Structural optimization has also been applied to acoustic design problems. Wadbro and Berggren [8] optimize an acoustic horn that efficiently radiates sound. Duhring et al. [9] use topology optimization for two types of problems. First, a room acoustic problem is considered where reflecting material is optimally distributed along the ceiling or the walls. Second, an outdoor sound barrier is optimized to reduce the sound power level behind the barrier. The same problem was considered by Greiner et al. [10] who applied shape optimization using genetic algorithms to optimize Y-shaped noise barriers. Shape optimization is also used by Abe et al. [11] to optimize noise barriers for railway viaducts and topology optimization is used by Kook et al. [12] to minimize the maximum main specific loudness.

Structural optimization is often found to lead to designs which are very sensitive to geometrical imperfections [13]. Since small variations in the design might result in a strongly deteriorated performance, robust optimization methods have been developed to take these uncertainties into account. Sigmund [14] and Wang et al. [15] proposed a robust optimization method to deal with manufacturing tolerances. A projection filter is added to the optimization to ensure a black and white design. The projection threshold is varied to simulate geometric imperfections and the worst performance of multiple designs originating from different projection thresholds is optimized. This methodology is applied to the elastodynamic problem in this paper to obtain designs that are both effective and robust with respect to geometrical imperfections.

This paper investigates the use of topology optimization for the design of two-dimensional wave barriers impeding wave transmission between a source and a receiver. This problem is encountered in problems of environmental vibrations, e.g. as generated by railway traffic [16]. The finite element method is used to determine the displacement field in the halfspace, with Perfectly Matched Layers (PMLs) at the boundaries to prevent spurious reflections. In order to minimize the transmission of waves, stiffened material is distributed in a design domain situated between the source and the receiver. A gradient-based optimization method is applied and the sensitivities are calculated using the adjoint method, making the calculation efficient for the large number of design variables considered. A worst case approach is adopted for obtaining a robust design with respect to geometrical imperfections where the worst performance of some (extreme) cases is optimized. This paper demonstrates the importance of considering robustness in the optimization process, and shows that the robust designs can be utilized as a source for simplified design solutions.

This paper is organized as follows. First, the method of topology optimization is briefly recapitulated and the optimization problem is presented. Next, wave barriers are designed with topology optimization for three types of excitation: harmonic point sources at a known frequency, harmonic point sources at a frequency situated in a given range, and broadband point sources. Finally, the sensitivity of the optimized design with respect to geometric imperfections is investigated and robust topology optimization is applied to obtain designs less sensitive to these uncertainties.

2. Formulation of the optimization problem

2.1. Finite element model

Fig. 1 shows the considered optimization problem. A two-dimensional homogeneous elastic half-space of material 1 is excited at the surface by a vertical point load. The aim is to minimize the response at an output point, located at the surface of the half-space. Therefore, a design domain is considered between the source and the receiver where a second stiffer material is introduced. The design domain has a cross-sectional area of 5×8 m^2 and is located at a distance of 5 m from the excitation point. The position where the performance is optimized is at another 5 m from the design domain, and, therefore, 15 m from the excitation point. The properties of the original material 1 and the stiffened material 2 are summarized in Table 1, with ρ being the mass density, C_p the longitudinal wave velocity, C_s the shear wave velocity, λ_p the wavelength of the longitudinal waves and λ_s the wavelength of the shear waves.

The elastodynamic problem is solved using the finite element method with two-dimensional four-node elements in plane strain. For the mesh, an element size of 0.25 m is used, corresponding to ten elements per shear wavelength λ_s at a frequency of 80 Hz, which is the upper limit considered in this paper. The discretization results in a displacement vector $\hat{\mathbf{u}}$ collecting N_{dof} degrees of freedom, determined by the following system of equations:

$$\hat{\mathbf{K}}\hat{\mathbf{u}} = \hat{\mathbf{p}} \tag{1}$$