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Dynamics of multilayer, multidisc viscoelastic rotor – An operator based higher order classical model ☆

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ABSTRACT

Inherent material damping plays a very significant role on dynamic behaviour of rotors. The material damping in a spinning rotor produces a tangential force along the whirl direction and its magnitude being proportional to spin speed. After certain value of spin speed, decided by the characteristic of the system, the tangential force becomes strong enough to throw the rotor centre out of the whirl orbit by inflating it progressively. This leads to destabilization of the system and corresponding speed is known as stability limit of spin speed. Stability limit of spin speed for Jeffcott rotor, by using viscous form of material damping model is straight forward and has been reported by several researchers, however the same analysis for viscoelastic material characteristics is not reported much. This analysis is very relevant for industrial requirements to replace bulky and heavy metal rotor by light but strong rotors. This is achieved either by reinforcing fibre or multi layering arrangements. Both of which are represented by viscoelastic constitutive behaviour. This paper gives mathematical derivation of equations of motion for multi-disc, multi-layered rotor–shaft-system. Both lumped mass and discretized approach (finite element) are presented here mathematically and numerical simulation results are compared. The lumped mass approach gives a concise yet acceptable accuracy of the results.

1. Introduction and research provocation

Rotating machines have various applications in our daily life; engines, generators, turbines, pumps, compressors, etc., are some examples of the same; Refs. [1–3] may be referred to display many such uses and applications. So, for efficient, safe and reliable functioning, vibration in rotating machinery must be as low as possible or should at best be contained within acceptable limits. Damping in non-rotating structures reduces vibratory motion by dissipating vibratory energy. However for rotors only stationary dissipative forces help to diminish vibration but rotating damping forces generated by material damping present in the spinning rotor may not always reduce rotor vibration. Rotating damping forces above certain speed,

☆ All color versions of the figures are available in online.

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Nomenclature

a, b	coefficients of material modulus
e	exponential
e	eccentricity
i	iota (imaginary unit)
l	length of element
n	number of coefficients in polynomial of modulus operator
$\{\mathbf{q}\}$	total degrees of freedom
u, v, w	mechanical displacement along the x, y and z axis respectively
r	radius of rotor
$\{\boldsymbol{u}\}$	excitation force
t	time in seconds
A	cross-sectional area
$[\mathcal{A}]$	system state matrix
$[\mathcal{B}]$	input matrix
$[\mathcal{C}]$	output matrix
$[\mathbf{C}]$	matrix, content all stiffness coefficients
$[\mathcal{D}]$	direct transmission matrix
D	diameter
D	first order differential time operator, i.e. $\frac{d()}{dt}$
$E()$	modulus operator
E	modulus of elasticity
$[\mathcal{E}]$	descriptor matrix
$\mathcal{F}$	objective function
$[\mathbf{G}]$	gyroscopic matrix
I	area moment of inertia
$\mathbf{I}$	identity matrix
J_P	polar moment of inertia
J_D	diametral moment of inertia
$[\mathbf{K}]$	stiffness matrix
L	length of rotor
L^*	disc position
$[\mathbf{M}]$	mass matrix
M	mass
N_d	number of disc
N_l	number of layer
$\{\mathbf{P}\}$	external nodal force vector
R	deformation of the rotor centre line
V	potential energy
$\{\mathcal{X}\}$	state vector
$\{\mathbf{y}\}$	output vector
φ, ϑ	rotation about y and z-axis respectively
$\mathfrak{R}$	radius ratio
σ	mechanical stress
ε	mechanical strain
ρ	mass density
ω	whirl speed
Ω	spin speed
η	loss coefficient

Subscript

cr	critical
d	disc
i	iteration
i, j, k	indices
l	layer
n	total number of degrees of freedom
r	rotor
D	diametral
P	polar
x, y, z	conventional coordinate axis
B	bending
C	circulatory

Superscript

e	element
T	transpose

Abbreviation

CM	classical model
FNF	first natural frequency
FEM	finite element model
MDF	modal damping factor
SLS	stability limit of spin speed
SWL	synchronous whirl line
UBR	unbalance response

Operators

()	operator
$(\cdot)$	order of differential equation
$(\sim)$	vector is in rotating coordinate
$(*)$	non-dimensional term
$(\hat{\ })$	assumed quantity

the limit of which is decided by the characteristics of the system, may add energy to the whirl orbit and result in destabilization of rotor–shaft systems. Despite this, for the need to have light yet strong and well damped rotor–shafts working primarily at normal temperature surroundings, composite material made of fibre reinforced or layer-wise construction are only alternatives. This paper attempts to obtain equations of motion of such a rotor–shaft-system by approximating rotor–shaft continuum as lumped masses as well as by finite beam elements. The former gives a simple yet effective prediction of the dynamic behaviour, whereas the latter is no doubt elegant but results in many equations of motion and is computationally intensive.

By and large every materials are understood to be viscoelastic, unlike elastic materials, strain produced in viscoelastic materials, causes simultaneous energy storage and dissipation when subject to dynamic loading. Here, stress at a point is not only proportional to strain but in general the stress and its derivatives are proportional to strain and their derivatives; the Hookean behaviour of a material, where stress is proportional to strain forms a special case of viscoelastic material. This