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Generalized warping effect in the dynamic analysis of beams of arbitrary cross section

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ABSTRACT

In this paper a general formulation for the nonuniform warping dynamic analysis of beams of arbitrary simply or multiply connected cross section, under arbitrary external loading and general boundary conditions is presented taking into account the effects of rotary and warping inertia. The nonuniform warping distributions are taken into account by employing four independent warping parameters multiplying a shear warping function in each direction and two torsional warping functions, respectively, which are obtained by solving the corresponding boundary value problems, formulated exploiting the longitudinal local equilibrium equation. A shear stress "correction" is also performed in order to improve the stress field arising from the employed kinematical considerations. Ten initial boundary value problems are formulated with respect to the displacement and rotation components as well as to the independent warping parameters and solved using the Analog Equation Method, a Boundary Element Method based technique in combination with an appropriate time integration scheme. The warping functions and the geometric constants including the additional ones due to warping are evaluated employing a pure BEM approach.

1. Introduction

In engineering practice, the dynamic analysis of beam-like members under flexure is frequently encountered. Besides, many conditions occurring in engineering practice, such as rotating machinery, wind and traffic loads, blast and earthquake forces, require taking into account inertial effects in the dynamic analysis of beams. The inclusion of such effects, especially through distributed mass models, which are considered to be more reliable, requires a rigorous analysis. In the vast majority of these cases, Euler–Bernoulli beam theory assumptions are adopted, and when shear deformation effect is non-negligible these assumptions are relaxed by using Timoshenko beam theory. However, both theories maintain the assumption that plane cross sections remain plane after deformation. By maintaining this assumption, the formulation remains simple; however, it fails to capture the well-known shear lag phenomenon, which was reported long ago (e.g., [1–3]) and observed in many structural members (e.g., beams of box-shaped cross section, folded structural members, beams made of materials weak in shear). Shear lag is associated with a significant modification of normal stress distribution, especially near the joint of the various cross-sectional components [4], due to non-uniform distribution of shear warping (Fig. 1d and e) [5].

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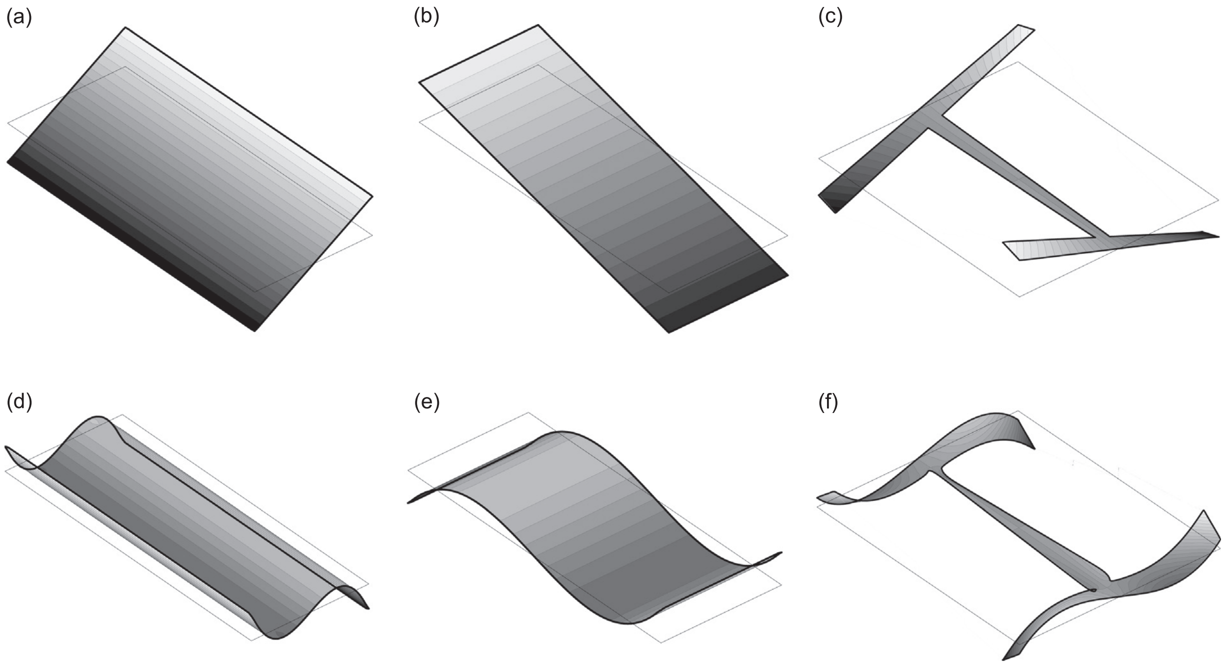

Fig. 1. Normal stress distribution due to flexure (a,b), primary torsional warping (c), shear warping (d,e) and secondary torsional warping (f).

In up-to-date regulations, the significance of shear lag effect in flexure is recognized; however, to simplify the analysis, the effective width concept [6–8] is recommended. This simplifying approach may fail to capture satisfactorily the actual structural behavior of the member, because the influence of the shear lag phenomenon is not constant along the beam length. Apart from the geometrical configuration of the cross section, it depends also on the type of loading [9]. Therefore, it is necessary to include the effects of nonuniform shear warping distribution in both the static and the dynamic analysis.

Generalizing the shear lag analysis problem, considerations similar to those made for flexure could be also adopted for the problem of torsion. It is well known that when a beam undergoes general twisting loading under general boundary conditions, it is led to nonuniform torsion. This problem has been extensively examined in the literature (e.g., [10,11]) and its major characteristic is the presence of normal stress due to primary torsional warping (Fig. 1c). In an analogy with Timoshenko beam theory, when shear deformation is of importance, the so-called secondary torsional moment deformation effect (STSDE) [12,13] has to be taken into account as well. Thus, it could be concluded that the additional secondary torsional warping due to STSDE (Fig. 1f) can cause effects similar to shear lag in flexure, i.e., a modification of the initial normal stress distribution. It is noted that because of the complicated nature of torsion, simplified concepts such as effective width cannot be applied to take this behavior into account.

Toward investigating shear lag effects, the inclusion of nonuniform warping in beam element formulations based on so-called higher-order beam theories [14] is of increased interest due to their important advantages over more refined approaches [15–17]. Beam elements are practical and computationally efficient and offer better insight into the structural phenomena, since they permit their isolation and independent investigation. Furthermore, because of easy parameterization of all necessary data, beam elements are more convenient for parametric analyses than more refined models which, in most cases, require the construction of multiple models.

The dynamic analysis of the generalized nonuniform warping problem has received relatively less attention than the static one. Some researchers have employed the assumptions of TTT (Thin Tube Theory) and the nonuniform torsion theory with an additional degree of freedom for nonuniform warping that is the rate of angle of twist [18–29] or an independent warping parameter [30,31]. Di Egidio et al. [32] describe the non-linear warping of open cross-section thin-walled beams in terms of the flexural and torsional curvatures and investigate the dynamics of thin-walled, open cross-section beams [33]. Minghini et al. [34] take into account shear strain effects due to both nonuniform bending and torsion in the analysis of thin walled cross sections. Some other researchers have employed the nonuniform torsion theory for beams of arbitrarily shaped cross sections with the rate of the angle of twist as an additional degree of freedom [35–42]. Sapountzakis and Tsipiras [43] have taken into account nonuniform torsion in the analysis of beams of arbitrarily shaped doubly symmetric cross sections using a secondary warping function. On the other hand Li et al. [44] developed an element that includes the effect of warping using a displacement field that is assumed to be cubic in the axial direction and quadratic in the transverse direction. Moreover, to the authors' knowledge, a BEM procedure for the dynamic analysis of the general nonuniform warping problem of beams of arbitrary cross sections has not been reported in the literature.

In this study, which is an extension of a previous work of the first two authors [45,46] at the dynamic regime, a general boundary element formulation for the dynamic nonuniform warping analysis of beams of arbitrary cross section, taking into