ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Thermoelastic damping effect of the micro-ring resonator with irregular mass and stiffness

Jung-Hwan Kim [a], Ji-Hwan Kim [b,*]

[a] Department of Mechanical and Aerospace Engineering, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul 08826, South Korea
[b] Institute of Advanced Aerospace Technology, Department of Mechanical and Aerospace Engineering, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul 08826, South Korea

ARTICLE INFO

Article history:
Received 2 January 2016
Accepted 13 January 2016
Handling Editor: L.G. Tham

Keywords:
Thermoelastic damping
Micro-ring
Multiple deviation

ABSTRACT

Fundamentally, vibration characteristic is a main factor for the stability of structures. In this regard, the irregularity of mass and stiffness distributions for the structure have been an interesting issue for many years. Recently, the Micro Electro Mechanical Systems (MEMS) are developed for various applications such as gyro sensors. In the present work, in-plane vibration of micro-ring structure with multiple finite-sized imperfections is investigated. Then, the unbalance of the structure is represented using Heaviside Step Function for the inextensional modeling of the ring. Also, thermoelastic damping (TED) due to internal friction is studied based on Fourier's one-dimensional heat conduction equation using Laplace Transform. To obtain the quality-factors (Q-factors) for imperfect micro-ring, analytical solutions are calculated from governing equations of motion with TED. And then, the natural frequencies and the Q-factors are observed to separate into lower and higher modes. Additionally, the vibration mode shapes are presented, and the frequency trimming concept due to attached imperfections is investigated.

1. Introduction

Based on a modern technology, the Micro Electro Mechanical Systems (MEMS) have been widely used as precision machine elements. For example, micro-ring gyro is commonly used for resonators as rate sensors. In this regard, thermoelastic damping (TED) effect is an important factor as well as elasticity property to be estimated in the micro- or nano-scale structure. Wang et al. [1] presented a hermetically encapsulated breathe-mode ring resonator with very high Q-factor. Wang et al. [2] investigated a novel-micro-machined vibration ring gyroscope with highly symmetric structure in order to improve the robustness at high and low temperatures. Prorok et al. [3] demonstrated a novel position-resolved resonance trimming strategy for silicon ring resonators. Pei [4] considered TED effect in a rotating micro-disk. Tai et al. [5] performed design, analysis, simulation and experiment on a ring gyroscope with piezoelectric effect. Guo [6] studied the effect on microbeam resonators using the generalized thermoelasticity theory. Guo et al. [7] evaluated geometrical influence on the damping in the micro-beam resonators using eigenvalue formulation. Tallur and Bhave [8] reviewed frequency scaling of Q-factors in a ring resonator. Kumar et al. [9] developed a model with tensile force and thermal conductivity in a clamped nano-resonator. Moosapour et al. [10] estimated the TED effect on the vibrations of thin beam resonators. Tai and Li [11] presented an

* Corresponding author. Tel.: +822 880 7383; fax: +822 887 2662.
E-mail address: jwhkim@snu.ac.kr (J.-H. Kim).

http://dx.doi.org/10.1016/j.jsv.2016.01.031

analytical model for TED in micromechanical resonators with thermodynamic parameter measuring the irreversibility in heat conduction. Joshi et al. [12] reviewed the strategies of design for controlling TED in nano- or micro-mechanical resonators. Peng et al. [13] discussed nonlinear dynamic analysis of a micro-actuator with nonlinear elasticity materials. Especially, Xi et al. [14] analyzed the Q-factor of the resonators including small trimming grooves for gyroscopes, and then the model is based on assumption as neglecting thermal profile due to finite-sized imperfections.

Generally, the TED effect is related with internal friction in the micro-structure, and then the energy dissipation results in inaccuracy of the device. Zener [15] analytically investigated the TED effect from the friction on microstructure. Lifshitz and Roukes [16] examined TED as a dissipation mechanism in MEMS and presented to design for lower energy loss. Yi and Matin [17] developed a finite element formulation to solve the problem in beam resonator systems with TED effect. Wong et al. [18] considered the application of Lifshitz's theory to thin circular ring model for in-plane flexural vibration. Parayil et al. [19] studied TED on the thick beam using Timoshenko beam theory by analytic and numerical methods.

Even for macroscopic structure, imperfections are inevitable for irregularity of mass and stiffness distributions. A lot of previous works concerned on the weight as the random point masses. Fox [20] considered flexural vibration of circular imperfect ring model with small attached masses and springs. Furthermore, Hong and Lee [21] predicted the local deviations on the in-plane vibration of nearly axisymmetric ring including finite mass and stiffness. And then, results of analytic simulations were compared with the experimental data. Park et al. [22] studied the effect of multiple local deviations on the mode pairs of the ring. Leizerovich and Taranukha [23] summarized the equations of circular shell and ring with small deviation on the frequency obtained by numerous methods.

In this work, micro-ring structure with finite sized mass and stiffness deviations including TED effect is modeled using Heaviside Step Function, and is solved analytically by Laplace transformation. In order to verify the analytical results of the ring model, frequency difference due to the imperfection is compared with the previous experimental data of macroscopic model. Furthermore, the Q-factors are obtained in order to relate the stability of motion. Finally, frequency trimming method is introduced to reduce the effect of deviation.

2. Micro-ring model with imperfections

Fig. 1 shows the micro-ring model with a rectangular-shaped cross-section including local mass and stiffness deviations. And then, the polar coordinate system is used in the analytical modeling.

2.1. Inextensional vibration with TED

Imperfect ring model is based on Euler–Bernoulli beam theory in thermal environment. Then, the total strain consists of bending stress and thermal expansion as the main variable in the in-plane vibration of the ring.

The relationship between circumferential stresses σ_θ and strains ε_θ can be expressed as [25]

$$\varepsilon_\theta = \frac{1}{E}\sigma_\theta + \alpha T_a \tag{1}$$

where E, α and T_a are isothermal Young's modulus, thermal expansion coefficient and ambient temperature, respectively.

Also, the equation can be rewritten in terms of σ_θ as

$$\sigma_\theta = E(\varepsilon_\theta - \alpha T_a) \tag{2}$$

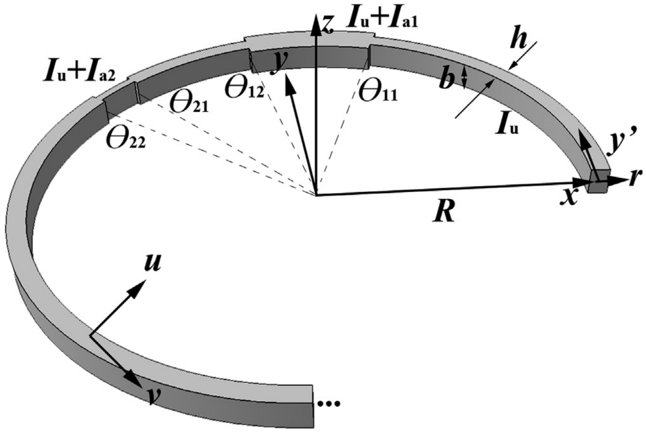

Fig. 1. Circular ring model with local deviations.

Please cite this article as: J.-H. Kim, & J.-H. Kim, Thermoelastic damping effect of the micro-ring resonator with irregular mass and stiffness, *Journal of Sound and Vibration* (2016), http://dx.doi.org/10.1016/j.jsv.2016.01.031