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A new method for analytical solution of inplane free vibration of rectangular orthotropic plates based on the analysis of infinite systems

S.O. Papkov

Department of Mathematics, Sevastopol State University, Sevastopol, Crimea

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ABSTRACT

In this paper, a new method for in-plane free vibration analysis of rectangular orthotropic plate is presented. The cases for completely clamped and completely free plates are considered in detail. The boundary value problem is essentially reduced to infinite systems of linear algebraic equations. New mathematical theory and associated results are presented for investigating the asymptotic behavior of non-trivial solution of quasi-regular infinite system. By enhancing the theory, an effective and accurate algorithm for determining the natural frequencies and modes shapes vibrations is developed. The accuracy and computational efficiency of the method are demonstrated by examples.

1. Introduction

Accurate determination of natural frequencies of plates for any arbitrary boundary conditions has always been a challenging problem, especially within the medium to high frequency range. Exact analytical solutions of the problem are known for rectangular plates with simple support boundary conditions of opposite edges. The investigation of this nature was mostly confined to free transverse vibration of rectangular plates. Surprisingly the first study for exact solutions of in-plane free vibration started later and was presented in 2006 by Gorman [1], though the Levy-type solutions for transverse vibrations was well known in middle of the 20th century. Although Gorman claimed that he was the first author to produce a thorough study of the exact solutions for the in-plane vibration, such type of exact solutions had already been presented by Wittrick and Williams [2] much earlier. Wittrick and Williams [2] not only obtained exact solutions, but also they derived the frequency dependent dynamic stiffness matrix for rectangular plate elements. The comparatively less and late attentions paid to in-plane free vibration of rectangular plates may be explained by virtue of inplane vibrational modes occur at relatively high frequencies when compared to that of the out of plane ones. Nevertheless, it is generally acknowledged that in-plane free vibration of plates can be very important in many practical problems of structural mechanics, particularly when thin plates are subjected to high speed tangential flow of a fluid, or when investigating the vibration transmission in built-up structures.

Bardell et al. [3] made a noteworthy contribution to the analysis of in-plane vibration of plates. The authors used Rayleigh–Ritz method for computing the natural frequencies of plates for various boundary conditions. Wang and Wereley [4] used Kantorovich–Krylov method to study different cases with mixed boundary conditions, particularly when the rectangular plate has some edges free and other edges clamped. The Fourier series method was employed by Du et al. [5] for plates with classical

E-mail address: stanislav.papkov@gmail.com

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boundary conditions and also for uniform elastically restrained edges. The superposition method was introduced by Gorman [6–9] for analysis of in-plane free vibration of rectangular plates. Accurate analytical type solutions were obtained for rectangular plates with classical edge conditions. The Ritz method using a set of trigonometric functions was employed by Dozio [10] to obtain accurate in-plane modal properties of rectangular plates with arbitrary non-uniform elastic edge restraints.

It should be noted, that the in-plane free vibration problem for rectangular plate is closely related to the free vibration problem of an infinitely long rectangular prism due to the dependence between plane-stress and plane-strain conditions. The relatively scarce number of publications devoted to this problem can now be significantly expanded. One of the pioneering works on this problem was presented by Mindlin and Fox [11], who investigated the harmonic oscillations of infinitely long bar of rectangular cross-section. Fromme and Leissa [12] reduced the free vibrations problem of parallelepiped to matrix eigenvalue problem with the help of variational approach. Grinchenko and Meleshko [13] developed the further modification of the superposition method by researching the plane strain problem for steady-state forced vibrations of rectangular isotropic parallelepiped. In fact they obtained analytic type solution and computed the natural frequencies of completely free rectangular prism (or analogously for plate for the plane stress problem).

Attempts to seek analytical type of solution in the form of series (truncated or un-truncated) with unknown coefficients for an arbitrary combination of boundary conditions have some coverage in literature. These attempts practically most of the time if not always lead to the final stage of solving a system of linear equations (finite or infinite) regardless of the use some variational method or method of superposition. Obviously, the usage of un-truncated series is more accurate from a mathematical point of view. However the problem of analysing and effectively solving the infinite system of linear algebraic equation is no-doubt difficult here. In a recent paper [14] a new effective method for analysis of such infinite systems and its application for free transverse vibration and buckling analysis of rectangular orthotropic plate was presented. This approach is based on the verification of existence of fixed point for operator defined by an infinite matrix with the help of derived criterion of regularity. The key ingredient in this method is the analytical evaluation of regularity of system after elimination some first unknowns.

In this paper the approach developed in Ref. [14] is applied to in-plane free vibration analysis of rectangular orthotropic plates for the determination of their natural frequencies. Besides, a new mathematical result allows to find all infinite set of unknown coefficients in series solution for in-plane free vibration problem. Therefore the method presented here gives the most accurate solution among other known solutions for plate with clamped and free edges.

2. General solution of the differential equations

In a right-handed rectangular Cartesian coordinate system with mutually perpendicular axes X, Y and Z, consider the in-plane free vibration of a rectangular orthotropic plate of sides $2a$ and $2b$ and thickness h. The plate is lying in the XY plane with its length ($2a$) and breadth ($2b$) are respectively parallel to the X and Y axes and the origin of the coordinate system is at the mid-point. Assume that the plane of elastic symmetry is located perpendicularly to the axis Z. Then the displacements components can be written as:

$$U(x,y,z,t)=u^0(x,y,t)+z\phi_y(x,y,t),\quad V(x,y,z,t)=v^0(x,y,t)-z\phi_x(x,y,t), W(x,y,z,t)=w^0(x,y,t) \tag{1}$$

The focus of this work being on the in-plane free vibration, the displacement components in the bending, namely $\phi_x(x,y,t)$, $\phi_y(x,y,t)$ and $w^0(x,y,t)$ can be omitted. The governing differential equations of free vibration for stress components have the same form as for an isotropic plate and can be written as:

$$\begin{aligned}\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}&=\rho\frac{\partial^2u^0}{\partial t^2}\\ \frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}&=\rho\frac{\partial^2v^0}{\partial t^2}\end{aligned} \tag{2}$$

In the case of an orthotropic material, the stress–strain relationships between displacements components and stresses can be set up with the help of the technical constants: Poisson's ratios ν_1 and ν_2 and Young's moduli E_1 and E_2 in the directions of axes X and Y respectively, and the shear modulus G

$$\begin{aligned}\frac{1-\nu_1\nu_2}{E_1}\sigma_{xx}&=\frac{\partial u^0}{\partial x}+\nu_2\frac{\partial v^0}{\partial y}\\ \frac{1-\nu_1\nu_2}{E_2}\sigma_{yy}&=\nu_1\frac{\partial u^0}{\partial x}+\frac{\partial v^0}{\partial y}\\ \frac{1}{G}\sigma_{xy}&=\frac{\partial u^0}{\partial y}+\frac{\partial v^0}{\partial x}\end{aligned} \tag{3}$$

For harmonic oscillation, i.e. when $u^0(x,y,t)=u(x,y)e^{i\omega t}$ and $v^0(x,y,t)=v(x,y)e^{i\omega t}$, $u(x,y)$ and $v(x,y)$ being the amplitudes of the displacements and ω the circular (or angular) frequency of oscillation, the substitution of Eq. (3) into Eq. (2) leads to

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