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Dispersion curve veering of longitudinal guided waves propagating inside prestressed seven-wire strands

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ABSTRACT

Elastic guided waves are of interest for the non-destructive evaluation of cables. Such structures are usually helical, multiwired and highly prestressed, which greatly complicates the understanding of wave propagation from a theoretical point of view. A remarkable feature is the occurrence of a missing frequency band in experimental time–frequency diagrams, sometimes referred to as notch frequency in the literature. The central frequency of this band increases under tensile loads. Recently, a numerical model has been proposed to compute the dispersion curves of prestressed helical seven-wire waveguides. Results have shown that the notch frequency indeed corresponds to a curve veering phenomenon between two longitudinal-like modes and that the increase of the notch under tensile loads is mainly due to interwire contact mechanisms. The main goal of this paper is to highlight the origin of this curve veering phenomenon, which is still unexplained up to the author's knowledge. This paper also provides further results which allow us to clarify the accuracy of numerical solutions as well as the influence of contact assumptions. First, the static part of the model, necessary to compute the prestress state including contact effects, is checked from reference analytical solutions. Owing to the importance of contact, the accuracy of results is discussed both in statics and in dynamics. The influence of slip contact conditions is outlined. Then, some numerical tests are conducted by varying the Poisson coefficient and the helix lay angle. These tests allows us to find out that the radial displacement constraint imposed on peripheral wires by the central one in the contact regions constitutes the main source of curve veering. More precisely, it is shown that a similar curve veering does occur for an uncoupled single peripheral wire when constrained by a radially blocked motion localized in its contact zone. Indeed, such a localized boundary condition completely breaks the circular symmetry of the wire cross-section, yielding coupling between longitudinal, flexural and torsional motion together with curve veering phenomena.

1. Introduction

Cables are widely used in modern constructions. In order to assess their structural health, the development of non-destructive evaluation techniques is necessary. Guided wave based methods are of particular interest because these waves can propagate over long distances, hereby increasing the potential distance of inspection. However, the understanding of mechanisms governing the propagation of guided waves is particularly complicated owing to the helical and multiwire

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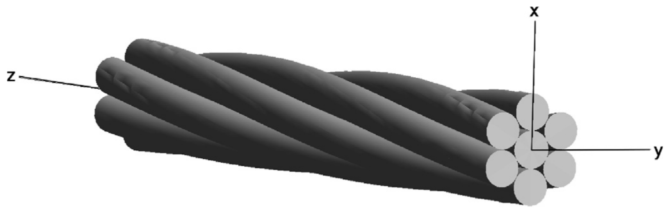

Fig. 1. Geometry of a seven-wire strand.

structure of strands, the basic elements constituting cables. The complexity of the problem is further increased by the presence of high tensioning forces applied on cables. These huge axial loads can influence the propagation of waves.

There exists a wide variety of strands. A common type, widely encountered in modern bridges, is the so-called seven-wire strand, made by one central cylindrical wire surrounded by six helical peripheral ones as sketched in Fig. 1. Experiments have shown that a seven-wire strand cannot generally be modeled as an effective cylinder [1–3]. More complex models of waveguides are hence needed. Recently, numerical models have been proposed in order to subsequently account for: the helical geometry of a single wire [4,5], the interwire coupling [6] and the effect of axial loads [7,8]. These models are based on a semi-analytical finite element (SAFE) method. Good agreement with experiments have been found on the so-called *notch* frequency and its increase under applied tensile loads. The *notch* frequency represents a missing frequency band experimentally observed in time–frequency diagrams [1,2] and is typical of strands. In Ref. [6], it has been shown that the *notch* frequency indeed corresponds to a curve veering phenomenon between the dispersion curves of two distinct longitudinal-like wave modes. Curve veering phenomena are generally found in eigenvalue problems of weakly coupled systems [9,10] and can be defined as the repulsion of two modal branches, veering away from each other instead of crossing. In Ref. [8], numerical tests have demonstrated that the notch frequency increase under tensile loads is indeed mainly influenced by interwire contact.

Nevertheless, some modeling works are still required in order to improve the model and the understanding of wave propagation inside strands. The modeling of contact needs to be checked with reference solutions and the origin of the curve veering phenomenon has not been explained yet. Besides, the convergence of numerical results has to be assessed, as it is well known in contact problems that a sufficiently fine mesh is required in contact regions to achieve an accurate modeling. The present paper aims to highlight these issues.

2. Theoretical background

2.1. SAFE modeling of helical waveguides under prestress

The SAFE method has been widely considered in Cartesian coordinates, for the analysis of straight waveguides (see for instance Refs. [11,12]). It has also been proposed in cylindrical coordinates for the study of toroidal waveguides [13,14].

The analysis of guided waves inside helical strands requires a specific curvilinear coordinate system, called twisting coordinate system. Such a system has constant non-zero torsion but zero curvature, and hence corresponds to a particular case of helical system. This coordinate system has been used for pretwisted beams [15]. Under the assumption of a linear elastic material, isotropic and homogeneous, it has been shown in Ref. [6] that a twisting system allows us to preserve translation invariance in seven-wire strands, which hereby yields a theoretical proof for the existence of guided waves in such structures. With this kind of system, the cross-section plane remains perpendicular to the straight axis but rotates around this axis by following peripheral wires. Since the central wire is circular and isotropic, the cross-section of the whole structure and its material properties remain translationally invariant in a twisting system. The torsion of the twisting system is given by $\tau_0 = 2\pi/L_0$, with L_0 denoting the helix pitch of peripheral wires under prestress. This section briefly reviews the main equations of the SAFE method written in a twisting system and including prestress effects. Further details can be found in Refs. [7,8].

Let us denote z the straight axis of the waveguide, fixed to the Cartesian system, (x,y) the cross-section twisting coordinates, k the axial wavenumber and ω the angular frequency. The application of a SAFE method consists in assuming an $e^{i(kz-\omega t)}$ dependence of acoustic fields before finite element (FE) discretization. Therefore, only the two-dimensional cross-section into the (x,y) plane of the structure has to be meshed. One points out that the e^{ikz} field dependence implies that axial variables must be separable from transverse variables in the governing equations of motion (this separation of variable is actually possible thanks to the proof of translational invariance along the z-axis that holds in a twisting system [6,16]).

The application of a SAFE method leads to the following matrix system governing wave propagation inside prestressed strands:

$$\{\mathbf{K}_{1\sigma} - \omega^2 \mathbf{M} + ik(\mathbf{K}_{2\sigma} - \mathbf{K}_{2\sigma}^{\mathrm{T}}) + k^2 \mathbf{K}_{3\sigma}\}\mathbf{U} = \mathbf{0}, \tag{1}$$