Journal of Sound and Vibration 367 (2016) 250–255

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

The spatial-matched-filter beam pattern of a biaxial non-orthogonal velocity sensor

Charles Hung Lee [a], Hye Rin Lindsay Lee [b], Kainam Thomas Wong [c,*], Mario Razo [d]

[a] Department of Mathematics, California State University, Fullerton, Fullerton, CA, USA
[b] Department of Mathematics, California State University, Fullerton, Fullerton, California, USA
[c] Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
[d] Department of Mathematics, California State University, Fullerton, CA, USA

ARTICLE INFO

Article history:
Received 27 May 2015
Received in revised form
20 December 2015
Accepted 23 December 2015
Handling Editor: Y. Auregan
Available online 19 January 2016

ABSTRACT

This work derives the "spatial matched filter" beam pattern of a "u–u probe", which comprises two uniaxial velocity sensors, that are identical, collocated, and oriented supposedly in orthogonality. This non-orthogonality may be *un*realized in real-world hardware implementation, and would consequentially cause a beamformer to have a systemic pointing error, which is derived analytically here in this paper. Other than this point error, this paper's analysis shows that the beam shape would otherwise be *un*changed.

1. Introduction

A *bi*axial velocity sensor (also called a "u–u probe") comprises two *uni*axial velocity sensors, each measuring the acoustic particle velocity along its axis. The *bi*axial particle velocity sensor has already been implemented [1–3], has been used for direction finding [4,5], and has its directivity and beam pattern investigated [6,7].

With the two constituent *collocated uni*axial velocity sensors orthogonally oriented relative to each other, for example, along the x-axis and y-axis respectively, their azimuth-parameterized array manifold [8,9] would equal

$$\mathbf{a}^{(2+0)}(\phi) = \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}, \tag{1}$$

in response to a emitter of point size, incident with unit power from either the far field or the near field, at an azimuth angle of $\phi \in [0, 2\pi)$, defined with respect to the positive x-axis, counterclockwise.

If this *bi*axial velocity sensor is further collocated with a (isotropic) pressure sensor (i.e. a hydrophone or a microphone), their far-field azimuth-parameterized array manifold becomes 3×1 and equals [8,9]

$$\mathbf{a}^{(2+1)}(\phi) = \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \\ 1 \end{bmatrix}. \tag{2}$$

* Corresponding author.
E-mail addresses: charleshlee@Exchange.FULLERTON.EDU (C.H. Lee), hyerin.lee@case.edu (H.R.L. Lee), ktwong@ieee.org (K.T. Wong), lilmario@csu.fullerton.edu (M. Razo).

http://dx.doi.org/10.1016/j.jsv.2015.12.046
0022-460X/

In the real world, a biaxial velocity sensor's two axes could only be approximately orthogonal, due to imperfect manufacturing in the factory and/or imperfect deployment in the field. Such a non-ideality would affect the biaxial velocity sensor's beam pattern.

Without any loss of generality, suppose that the y-axis remains correctly aligned, but the x-axis is misoriented by an angle of $\tilde{\phi}$, to form a new $\tilde{x}$-axis on the x–y plane. Refer to Fig. 1. Then, this non-orthogonal $\tilde{x}$–y biaxial velocity sensor's 2×1 array manifold $\tilde{\mathbf{a}}^{(2+0)}(\phi, \tilde{\phi})$ may be related to the ideally orthogonal $\mathbf{a}^{(2+0)}(\phi)$ through a 2×2 transformation $\mathbf{R}^{(2+0)}$ as follows:

$$\tilde{\mathbf{a}}^{(2+0)}(\phi, \tilde{\phi}) = \underbrace{\begin{bmatrix} \cos(\tilde{\phi}) & -\sin(\tilde{\phi}) \\ 0 & 1 \end{bmatrix}}_{=\mathbf{R}^{(2+0)}(\tilde{\phi})} \mathbf{a}^{(2+0)}(\phi). \tag{3}$$

Similarly, a non-orthogonal 3×1 $\tilde{\mathbf{a}}^{(2+1)}(\phi, \tilde{\phi})$ may be related to the ideally orthogonal $\mathbf{a}^{(2+1)}(\phi)$ through a 3×3 transformation $\mathbf{R}^{(2+1)}(\tilde{\phi})$ as follows:

$$\tilde{\mathbf{a}}^{(2+1)}(\phi, \tilde{\phi}) = \underbrace{\left[\begin{array}{cc:c} \multicolumn{2}{c:}{\multirow{2}{*}{$\mathbf{R}^{(2+0)}$}} & 0 \\ & & 0 \\ \hdashline 0 & 0 & 1 \end{array}\right]}_{=\mathbf{R}^{(2+1)}(\tilde{\phi})} \mathbf{a}^{(2+1)}(\phi). \tag{4}$$

Suppose that a "spatial matched filter" beamformer aims to focus a non-orthogonal biaxial velocity sensor (with or without a pressure sensor) toward a "look direction" of $\phi = \phi_L$, but the beamformer is unaware of the biaxial velocity sensor's possible non-orthogonality. Then, the beam pattern would equal

$$B^{(2+j)}\left(\phi, \tilde{\phi}, \phi_L\right) = \frac{\left(\mathbf{a}^{(2+j)}(\phi_L)\right)^T \mathbf{R}^{(2+j)}(\tilde{\phi}) \mathbf{a}^{(2+j)}(\phi)}{\max_{\phi}\left[\left(\mathbf{a}^{(2+j)}(\phi_L)\right)^T \mathbf{R}^{(2+j)}(\tilde{\phi}) \mathbf{a}^{(2+j)}(\phi)\right]}, \quad \forall j = 0, 1, \tag{5}$$

where the superscript ^{T} denotes the transposition operator.

This paper will derive and will analyze the "spatial matched filter" beam patterns of the non-orthogonal $\tilde{\mathbf{a}}^{(2+0)}(\phi)$ in Section 2.1, and the non-orthogonal $\tilde{\mathbf{a}}^{(2+1)}(\phi)$ in Section 2.2.

2. The "spatial matched filter" beam pattern under *non*-orthogonality

To facilitate the subsequent derivation, recall the following well known fact:

Proposition 1. *Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be arbitrary vectors in $\mathbb{R}^N$, with $\mathbf{v}_2$ having a Euclidean norm of $\|\mathbf{v}_2\| = 1$; and let $\mathbf{R}$ be an $N \times N$ matrix. For any specific $\mathbf{v}_1$, the scalar function $F(\mathbf{v}_2) := \mathbf{v}_1^T \mathbf{R} \mathbf{v}_2$ would maximize to $F(\mathbf{v}_2 = \mathbf{v}_2^o) = \left\|\mathbf{R}^T \mathbf{v}_1\right\|$, where $\mathbf{v}_2^o := \frac{\mathbf{R}^T \mathbf{v}_1}{\left\|\mathbf{R}^T \mathbf{v}_1\right\|}$*

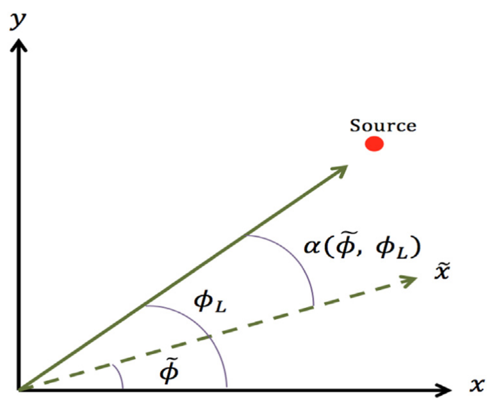

Fig. 1. The non-orthogonality angle $\tilde{\phi}$ rotates the x coordinate, to give the non-orthogonal $\tilde{x}$–y coordinates.