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Equivalent bilinear elastic single degree of freedom system of multi-degree of freedom structure with negative stiffness

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ABSTRACT

The advantage of negative stiffness (NS) is to reduce the structural internal force, especially base shear, which is necessary to be considered during structural design. During design procedures, the structural response should be estimated. For the purpose of estimating the response of the structure with NS, this paper presents an innovative approximate method to transfer a multi degree of freedom (MDOF) structure with NS in the 1st story (MNS1) to an approximate single degree of freedom (SDOF) bilinear elastic system (ASBS). The accuracy of approximation from the MNS1 to the ASBS is studied by the sensitivity method of mode shapes in which the relative changes in structural mode shapes are related to the variation of stiffness. An eight-story numerical example is employed to illustrate the accuracy of approximation by the proposed method. The results show that the accuracy of approach is acceptable, and ASBS can estimate the response of the MNS1.

1. Introduction

For the earthquake engineering, the most important function of structural stiffness is to resist ambient excitation and prevent structural inter-story deformation. Although the larger stiffness can resist the more external force, the inter-story shear force would be increased with the increasing of stiffness[1,2]. The concept of stiffness weakening was proposed by Reinhorn et al. [3] and Viti et al. [4], in which the internal force or acceleration is attenuated by the reducing of structural stiffness. There is some research on the isolator with negative stiffness for the application of vehicles, whose negative stiffness induces the stiffness weakening [5,6]. However, if the structure is actually weakened by reducing the strength of beams (or in some cases beams and columns), it can result in early yielding and permanent deformation, which would be very dangerous for structural safety. Antoniadis et al. [7]. proposed a novel vibration damping and isolation concept using negative stiffness, which did not decrease the overall structural stiffness with embedded negative stiffness elements, and counteracted the inertial and excitation forces by the proper redistribution and reallocation of the stiffness and the damping elements of the system. Nagarajaiah et al. [8] proposed the “apparent weakening” that mimics the early “yielding” without inducing the structural inelastic behavior, and makes the structure remain elastic. The realization of the “apparent weakening” can be performed by the negative stiffness (NS), which can be generated by the negative stiffness device (NSD) [9–11].

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In order to make sure the safety of a structure, the structural response should be estimated during structural design. Due to the advantage of NS to reduce the structural internal force, it is necessary to consider the NS property during estimating structural response. Although time-history analysis of an elaborate analytical model is probably one of the best options, its applicability requires large computational time and effort for establishing elaborate model and for the estimation of a structural response under seismic loading condition. In civil engineering, many building codes adopted equivalent single-degree-of-freedom (SDOF) systems to evaluate the overall performance of the structure, which do not need to perform the time-consuming computation for the elaborate analytical model. Most of multi-degree-of-freedom (MDOF) structures are always regarded to be linear elastic, which can be converted to SDOF system easily by mode superposition method [12]. However, a MDOF structure with NS that exhibits nonlinear property is hard to be transferred to an equivalent SDOF system. There are several studies on this conversion for the nonlinear system. Chopra and Goel [13] proposed the mode pushover analysis (MPA) method which decomposes the structure according to the mode superposition, and use the static pushover to determine the yielding point for each mode. They also pointed out that the MPA lacks a rigorous theory but it is simple and effective in most cases for structural designs. Makarios [14] defined an equivalent nonlinear SDOF system to represent the MDOF structure through the pushover procedure. Tjhin et al. [15]. presented an energy-based pushover approach, which established a more accurate capacity curve to provide a better estimation of the peak roof displacement. Vamvatsikos and Cornell [16] exploited the static pushover and incremental dynamic analysis to obtain an accurate estimate. Pasala et al. [17] investigated the SDOF building with the NSD and its linear or bilinear elastic behavior. Ray et al. [18] studied the nonlinear inelastic behavior of the NSD in the SDOF system. Pasala et al. [19] have studied MDOF with NSD in first floor. Yang et al. [20] studied the dynamic and power flow behavior of a nonlinear vibration isolation system with the negative stiffness mechanism, which can enlarge the frequency band for the effective vibration isolation. Zou and Nagarajaiah [21]. investigated the piecewise linear dynamic oscillator with negative stiffness followed by positive stiffness, which derived approximate the periodic solutions. The conversion from the MDOF system with the NS to the SDOF, which is required for the design of real engineering structures, has not currently been studied with related research.

This paper presents a new approximate method to convert the bilinear elastic MDOF to an equivalent SDOF. The MDOF structure with the NS only in the 1st story (MNS1) is considered, which is quite effective in reducing the base shear and displacement. Based on the mode superposition method [12], the conversion is derived to obtain the approximate SDOF bilinear elastic system (ASBS). The accuracy of approximation from the MNS1 to the ASBS is analyzed by the sensitivity method for mode shapes proposed by Zhao and DeWolf [22] who investigated the relative changes in the structural mode shape due to the changes in the structural stiffness. An eight-story numerical example is employed to illustrate the accuracy of approximation by the proposed method. Different cases, such as different ground motions, different NS parameters, and the structures with different natural frequencies, are considered to exhibit the effectiveness of the proposed method.

It should be noted that the NS could increase the structural deformation in the floor and result in the inelastic behavior of structure, which can be prevented by the supplemental damping. However, the study in this paper is the basic research for the negative stiffness, which only focuses on the elastic behavior without the supplemental damping.

2. Model definition

Different from the restoring force provided by the positive structural stiffness against the deformation, the negative stiffness "restoring force" assists deformation. The original stiffness and ideal negative stiffness are shown in Fig. 1 (a) and (b).

The negative stiffness can weaken the structure when added to the original structural stiffness. However, it would be helpful to guarantee the stability of structures to maintain the original stiffness under some small excitation [17]. Thus, the practical negative stiffness will have the force-deformation relation as in Fig. 1(c). The combination of the original stiffness and the negative stiffness in Fig. 1(c) is shown in Fig. 2.

Considering an n degree-of-freedom shear-frame structure, the dynamics can be described by a second-order differential equation as follows

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=-\mathbf{M}\mathbf{I}\ddot{x}_g \tag{1}$$

where $\ddot{x}_g$ is the earthquake ground acceleration; $\mathbf{I}\in\Re^{n\times 1}$ is the vector whose element value is equal to 1; $\mathbf{M},\mathbf{C},\mathbf{K}\in\Re^{n\times n}$ are

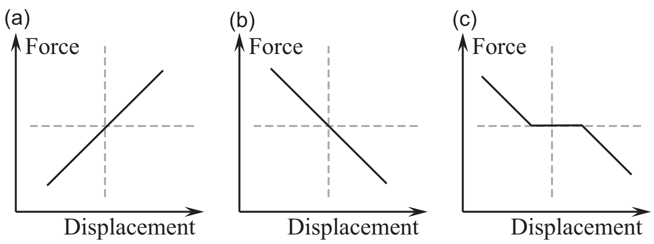

Fig. 1. Schematic diagrams depicting the force-deformation behavior: (a) positive stiffness; (b) negative stiffness; and (c) negative stiffness with gap.