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Modal density of thin composite cylindrical shells

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ABSTRACT

Modal density is an important parameter in Statistical Energy Analysis (SEA) based response estimation. Many space structures use composite cylinders. Modal densities of such structural elements are not reported. In this work an expression for modal density of composite cylindrical shells is derived. Its characteristics and sensitivity to various parameters are discussed. The frequency at which the modal density has a maximum is derived. Modal densities of typical composite cylinders are obtained. It is shown that computing modal density considering an equivalent isotropic cylinder can lead to significant errors.

1. Introduction

Estimation of responses of structures to high frequency dynamic excitation is normally carried out using a technique called Statistical Energy Analysis (SEA) [1,2]. One of the important parameters required for SEA based calculations is the number of resonant modes present in a frequency band. To be useful for SEA based calculations, the number of modes has to be estimated using a closed form expression and cannot be through a method like Finite Element method. Therefore expressions for modal densities of several structural forms are derived and are in use [3–5].

Many spacecraft structures have a central cylinder which is a cylindrical shell made of composite material. There are several studies reported on the modal densities of cylindrical shells. Heckl [6] obtained the modal density of a thin cylinder for the out of plane motion. In his work, natural frequency was obtained as the frequency at which the impedance vanished and the number of modes was estimated from the maximum value of half waves possible below the given frequency. Another work on modal densities of shells is by Bolotin [7] in which the modal densities of thin shells are obtained using wavenumber diagrams and expressed the modal density in terms of elliptical integrals. Szechenyi [8] obtained the modal density of a thin cylinder by measuring the area in the wavenumber plane and proposed empirical relations for its estimation. Maymon [9] presented the modal densities of stringer stiffened shells by adding the modal densities of monocoque cylinder with those of stiffeners.

Modal densities of sandwich cylinders are also reported. Wilkinson [10] derived an expression for modal densities of sandwich cylinders incorporating shear deformation of the core and Erickson [11] modified the expression considering rotary inertia. Ferguson and Clarkson [12] obtained an expression for estimating modal density of paraboloidal structural element. Elliot [13] presented expressions for the modal densities of thin as well as honeycomb sandwich cylindrical shells in the form of integrals which were evaluated numerically.

All the above works are on the isotropic shells or honeycomb sandwich shells with isotropic face sheets and no expressions are reported for composite cylindrical shells.

In this work an expression for modal density of composite cylinder is derived. For deriving the expression for modal density, an expression for the natural frequency is required. A closed form expression for frequency of orthotropic cylindrical shells is reported by Soedel [14]. This expression is based on Donnell's shallow shell theory where shear deformation and

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Nomenclature

u_x, u_θ, u_r	displacement along the longitudinal, tangential and radial directions
$N_{xx}, N_{\theta\theta}, N_{x\theta}$	force resultants per unit length
$M_{xx}, M_{\theta\theta}, M_{x\theta}$	moment resultants per unit length
$Q_{xr}, Q_{\theta r}$	shear forces per unit unit length
q_x, q_θ, q_r	external forces per unit area.
ρ_m	mass per unit area
ρ_v	density of the material
$\varnothing$	Airy's stress function
A_{ij}	extensional stiffness terms
B_{ij}	coupling stiffness terms
D_{ij}	bending stiffness terms
a	radius of the cylindrical shell
h	thickness of the shell
L	length of the cylindrical shell
A	Surface area of the cylindrical shell
K_1, K_2	function of wavenumbers
m	axial half wavenumber
n	circumferential full wavenumber
$n(\omega)$	number of modes per rad/s
$n(f)$	number of modes per Hz
$N(\omega)$	number of modes below the radian frequency 'ω'
$N(f)$	number of modes below the cyclic frequency 'f'
f_s	frequency of maximum modal density

rotary inertia are neglected. First the expression for natural frequency is derived. Expression for the modal density is then derived following an approach similar to those for isotropic shells. To validate the expression, mode count obtained using this expression is compared with the number of modes computed by a Finite Element Model using NASTRAN. Characteristics of modal densities of composite cylinders are discussed. Modal densities of typical composite cylinders are then presented. Modal densities if calculated using the expression for isotropic shells with equivalent isotropic properties are discussed.

2. Governing differential equations and natural frequency

There are several theories for describing the elastic behavior of the shells. In the present work, Donnell's shallow shell theory [14] is considered. The coordinate axes are longitudinal (x), tangential (θ) and radial (r) as shown in Fig. 1. The displacement along the longitudinal direction is u_x, along the tangential direction (linear displacement) is u_θ and along the radial direction is u_r. Let the radius of the shell be 'a' and the length be 'L'.

2.1. Assumptions

The assumptions used in arriving at the differential equations are summarized below.

1. The shell is cylindrical.
2. The shell is thin, i.e., thickness of the shell is much less compared to the radius ($\frac{h}{a} \ll 1$) (Love's assumption).
3. Plane stress condition exists.
4. $\varepsilon_r = 0$, i.e., the displacement u_r is independent of z.
5. The transverse planes that are normal to the un-deformed layers remain plane and normal after deformation (Kirchoff–Love assumptions). This means the shear deformations are negligible.
6. Donnell–Vlasov theory is adopted. Influence of inertia force in the in-plane direction is neglected. This is restricted to normal loading.
7. Material is linearly elastic.
8. The laminate is symmetric.
9. The laminate is balanced.
10. The laminate is specially orthotropic.
11. Mass distribution is uniform, i.e. mass per unit area is constant.
12. Rotary inertia is neglected.
13. The displacements u_x and u_θ are not independent but related by Airy's stress function.

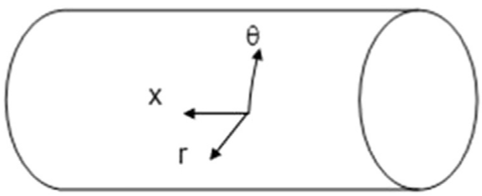

Fig. 1. Coordinate system.