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# Free transverse vibration of a wrinkled annular thin film by using finite difference method

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A B S T R A C T

This paper investigates the free transverse vibration of a wrinkled annular thin film. The non-dimensional Hamilton motion equation of the wrinkled annular thin film is established, which is solved by using the finite difference method to acquire the vibration frequency and mode. The predicted vibration characteristics are verified by the experimental measurements based on the digital image correlation (DIC) technique. The results show that wrinkles have great effects on the vibration of the annular thin film. Especially for the heavily wrinkled cases, the local–global interactive mode dominates the vibration of the annular thin film. The frequency increases as the wrinkling level increases which is mainly due to the increased nonlinear geometric stiffness. The results provide favorable supports for understanding the role of nonlinear wrinkling on the vibration of thin films.


## 1. Introduction

The problem of determining the natural frequency and mode of a thin film is an important component in the design of many engineering devices. Examples include architectural and civil structures, nano/micro electromechanical systems, and space-based applications such as radio antennas and optical reflectors, etc [1–5].

Dozens of theoretical studies on thin film vibration exist in the literature, which cover linear and nonlinear models, various shapes and boundary configurations, and numerous analysis methods including closed-form, asymptotic expansions, and numerical methods (FEM, FDM, etc.), etc [6–12]. Such studies are ongoing and of current interest.

However, little attention has been paid to the theoretical studies on the vibration of a wrinkled thin film, which is probably due to the lack of an efficient analytical method for solving the mathematical challenges [13,14]. Currently, several vibration simulations of a wrinkled thin film have provided us with some promising results to understand the effects of wrinkles on the vibration frequency [15–20]. Despite this progress, the interactive nature between wrinkles and vibration mode is still unclear.

In this work we are mainly concerned with the theoretical analysis of vibration frequency and mode of a wrinkled annular thin film. In Section 2, the non-dimensional Hamilton motion equation of the wrinkled annular thin film is established, which is solved by using the finite difference method to acquire the vibration frequency and mode. In Section 3,

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**Nomenclature**

| | |
|---|---|
| $r_1$ | inner radius |
| $r_2$ | outer radius |
| $E$ | Young's modulus |
| $h$ | thickness |
| $\nu$ | Poisson's ratio |
| $u_0$ | initial displacement |
| $M$ | torque |
| $\nabla^4$ | Laplace operator |
| $\rho$ | radial parameter $(r/r_2)$ |
| $\overline{m}$ | density per unit area |
| $\eta$ | ratio $(r_1/r_2)$ |
| $D$ | bending stiffness |
| $a_i$,$b_j$ | fit coefficients |
| $(\rho,\phi)$ | polar coordinates |
| $w(r,\varphi)$ | out-of-plane displacement |
| $\Phi(r,\varphi)$ | stress function |
| $\overline{w}(\rho,\varphi)$ | non-dimensional displacement |
| $\overline{\Phi}(\rho,\varphi)$ | non-dimensional stress function |
| $\overline{w}^w(\rho,\varphi)$ | post-wrinkling deflection |
| $\overline{\Phi}^*$,$\overline{\Phi}^{**}$ | general and particular solutions |
| $\overline{\sigma}^*_{\alpha\beta}(\rho,\varphi)$ | pre-buckling stress state |
| $\overline{\sigma}^w_{\alpha\beta}(\rho,\varphi)$ | post-wrinkling stress state |
| $\overline{\sigma}^{**}_{\alpha\beta}(\rho,\varphi)$ | stress state induced by wrinkles |
| $\tilde{\overline{w}}^w_t$ | deflection due to the vibration |
| $\hat{\overline{w}}^w(\rho,\varphi)$ | vibration waveform function |
| $\omega^w$ | circular frequency of wrinkled film |
| $p_1$,$p_2$ | the highest order of basis functions |

non-contacted experimental measurements are conducted on the vibration of the wrinkled annular thin film to verify the prediction accuracy. In Section 4, the role of wrinkles on the vibration characteristics of the annular thin film is elucidated.

## 2. Vibration of a wrinkled annular thin film

At first, a model of an annular thin film under inner torsion and outer tension is considered, as illustrated in Fig. 1. The inner and outer radii of this annular thin film are marked as $r_1$ and $r_2$, respectively. The outer boundary is stretched initially which results in a uniform displacement field of $u_0$. Meanwhile, a torque $M$ is applied on the inner boundary. A cylindrical system of coordinates $(r,\phi,z)$ is used to define various parameters associated with this problem.

### 2.1 Post-wrinkling stress and deflection

Wrinkling is generally regarded as a typical nonlinear out-of-plane deflection which can be derived from the non-dimensional large deflection von Kàrmàn equations, which are shown as follows [21]:

$$\nabla^4\overline{w}=\chi\left[\overline{\sigma}_{\rho\rho}\frac{\partial^2\overline{w}}{\partial\rho^2}+2\overline{\sigma}_{\rho\varphi}\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\overline{w}}{\partial\varphi}\right)+\overline{\sigma}_{\varphi\varphi}\left(\frac{1}{\rho}\frac{\partial\overline{w}}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\overline{w}}{\partial\varphi^2}\right)\right], \tag{1-a}$$

$$\nabla^4\overline{\Phi}=\left[\left(\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\overline{w}}{\partial\varphi}\right)\right)^2-\frac{\partial^2\overline{w}}{\partial\rho^2}\cdot\left(\frac{1}{\rho}\frac{\partial\overline{w}}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\overline{w}}{\partial\varphi^2}\right)\right], \tag{1-b}$$

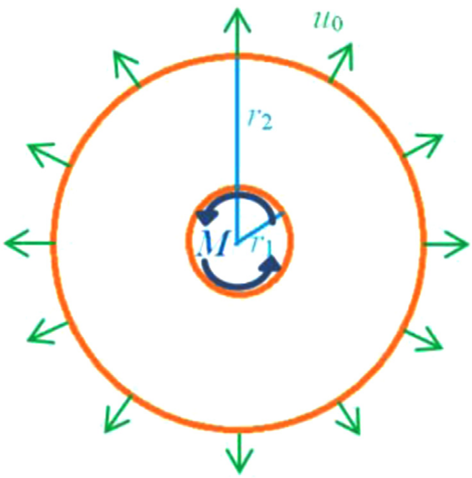


**Fig. 1.** A model of an annular thin film under inner torsion and outer tension.

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