Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Multi-modal vibration amplitudes of taut inclined cables due to direct and/or parametric excitation

J.H.G. Macdonald *

Department of Civil Engineering, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, UK

ARTICLE INFO

Article history:
Received 19 September 2014
Received in revised form
3 November 2015
Accepted 6 November 2015
Handling Editor: W. Lacarbonara

Keywords:
Cable dynamics
Inclined cables
Parametric excitation
Nonlinear vibrations
Multi-mode model
Modal coupling
Cable sag

ABSTRACT

Cables are often prone to potentially damaging large amplitude vibrations. The dynamic excitation may be from external loading or motion of the cable ends, the latter including direct excitation, normally from components of end motion transverse to the cable, and parametric excitation induced by axial components of end motion causing dynamic tension variations. Geometric nonlinearity can be important, causing stiffening behaviour and nonlinear modal coupling. Previous analyses of the vibrations, often neglecting sag, have generally dealt with direct and parametric excitation separately or have reverted to numerical solutions of the responses. Here a nonlinear cable model is adopted, applicable to taut cables such as on cable-stayed bridges, that allows for cable inclination, small sag (such that the vibration modes are similar to those of a taut string), multiple modes in both planes and end motion and/or external forcing close to any natural frequency. Based on the method of scaling and averaging it is found that, for sinusoidal inputs and positive damping, non-zero steady state responses can only occur in the modes in each plane with natural frequencies close to the excitation frequency and those with natural frequencies close to half this frequency. Analytical solutions, in the form of non-dimensional polynomial equations, are derived for the steady state vibration amplitudes in up to three modes simultaneously: the directly excited mode, the corresponding nonlinearly coupled mode in the orthogonal plane and a parametrically excited mode with half the natural frequency. The stability of the solutions is also identified. The outputs of the equations are consistent with previous results, where available. Example results from the analytical solutions are presented for a typical inclined bridge cable subject to vertical excitation of the lower end, and they are validated by numerical integration of the equations of motion and against some previous experimental results. It is shown that the modal interactions and sag (although very small) affect the responses significantly.

1. Introduction

Cables are used as structural elements in various contexts at various scales and are often prone to quite large transverse vibrations due to their low inherent damping. Cable dynamics can be complex and has attracted extensive research, especially regarding the effects of geometric nonlinearities, as reviewed by Nayfeh and Pai [1], Rega [2,3] and Ibraim [4]. The dynamic behaviour of cables is greatly influenced by the static sag, as addressed by Irvine and Caughey [5] and quantified by

* Tel.: +44 117 331 5735.
E-mail address: john.macdonald@bristol.ac.uk

http://dx.doi.org/10.1016/j.jsv.2015.11.012
0022-460X/© 2015 Published by Elsevier Ltd.

Irvine's parameter λ^2, whereby the sag increases the natural frequencies in the plane of the sag (in-plane) relative to those of a taut string or the cable in the out-of-plane direction. Much of the past research has been on taut strings (i.e. no sag, $\lambda^2=0$) or cables in the crossover regions, the first of which occurs when $\lambda^2=4\pi^2$. At crossovers the natural frequency of an odd (symmetric) in-plane mode has been increased sufficiently by the sag that it equals that of an even (anti-symmetric) mode in each plane, leading to strong modal interactions [6–9]. This situation is relevant to cables with significant sag such as electricity transmission lines. In contrast this paper deals with the vibration amplitudes of taut cables, i.e. those with very small sag (of which taut strings are a special case), with $\lambda^2 \leq O(1)$. This applies, for example, to the inclined cables of cable-stayed bridges. For these cables the natural frequencies and mode shapes are close to those for a taut string, i.e. a harmonic series, the sag causing just slight detuning of the natural frequencies of the odd in-plane modes relative to the corresponding out-of-plane modes. This then leads to interactions of different combinations of modes to those for cables in the crossover regions. At crossovers there are three modes with the same natural frequency (two in-plane and one out-of-plane) and one out-of-plane mode with half the natural frequency, whereas for taut cables there are two modes (one in each plane) with close natural frequencies and, where those are even modes, two more (also one in each plane) close to half the natural frequency.

The main aim of this paper is to provide analytical solutions for the steady state amplitudes of vibrations of taut inclined cables due to harmonic planar excitation, for different combinations of modes that may be directly and/or parametrically excited and which interact with each other nonlinearly. A secondary aim is to identify how very small sag ($0<\lambda^2\leq O(1)$) causes differences in the behaviour from that of a taut string.

Cables may be excited dynamically by external forcing, such as from wind loading, or by motion of one or both of the ends, such as from bridge deck or tower motion. Often in practice, in accordance with design guidelines [10,11], the components of end motion transverse and axial to the cable axis are considered separately, the former providing direct excitation similar to external forcing and the latter giving parametric excitation via tension variations. However, in some situations, such as in cable-stayed bridges or guyed masts, there can be cable end motion with components both axial and transverse to the cable, providing both types of excitation simultaneously. The resulting behaviour can be affected significantly by nonlinear interactions between the modal responses. Although strictly there can be dynamic interactions of the cable with the rest of the structure [12] here they are neglected and the cable end motions are prescribed, which is a reasonable representation of the behaviour when the rest of the structure is suitably massive.

For direct excitation only, of a taut string subject to harmonic planar excitation, Nayfeh and Mook [13] identified planar and non-planar steady-state solutions using the method of multiple scales. The amplitude of the planar solution was found to be unstable above a critical amplitude. Thereafter a non-planar whirling solution was stable, the amplitudes of the in-plane and out-of-plane motions being found by numerical continuation. Miles [14] similarly derived the averaged response of a 2-degree-of-freedom (2DOF) model of a taut string (one mode in each plane with the same natural frequency) subject to external direct excitation. He also considered stability of the solutions and identified Hopf bifurcations of the non-planar response. Bajaj and Johnson [15] discuss the solutions further, including periodic and chaotic solutions.

Meanwhile, considering responses to external forcing of a planar 1DOF model but allowing for sag, Benedettini and Rega [16] used a fourth order multiple time scale perturbation method to identify the steady state response amplitudes. It was shown that a suspended cable with $\lambda^2=15.36$ exhibited initial softening behaviour due to the quadratic nonlinearity and subsequent stiffening from the cubic nonlinearity, but a taut cable with $\lambda^2=1.536$ exhibited only stiffening behaviour from the cubic term, with negligible effect of the quadratic term. Later Gattulli et al. [17] compared numerical continuation of a 10-mode analytical model and numerical integration of a finite element formulation for an externally forced cable with $\lambda^2=15.36$ close to a 2:1 resonance between the natural frequencies of the third and first in-plane modes, and for a slacker cable with $\lambda^2>4\pi^2$. The two methods gave similar results and although complex modal interactions were identified it was found that only up to three low frequency modes, with certain resonant conditions, were significantly involved in the solutions. It was also found that the kinematic condensation of the longitudinal displacements in the analytical model gave no appreciable error. For cables with a larger range of Irvine's parameter, Srinil and co-authors [18,19] considered the effects of sag, inclination and kinematic condensation on modal interactions around frequency crossovers and 2:1 resonances that occur for certain values of λ^2 above 15. For such cables the inclination causes a variation in tension over the length giving an asymmetric static sag profile, leading to frequency avoidances, rather than crossovers, and gives coupling between more modes and increased significance of the longitudinal displacement than for horizontal cables. However, for taut cables the variation in tension over the length and the associated features are negligible.

For parametric excitation alone, there is zero response until the input amplitude exceeds a certain threshold, dependent on the damping, after which large amplitude transverse vibrations of the cable occur, which are affected relatively little by the damping. Different authors have calculated the stability boundaries and the amplitudes of vibrations based on the Mathieu–Hill equation for a single mode [20–22]. Uhrig [21] included nonlinear coupling terms between modes in his equations of motion but reduced them to uncoupled linear Mathieu equations to define the stability boundaries.

A situation in which both direct and parametric excitation can occur is an inclined taut cable subject to vertical or horizontal support excitation of one end, which has been considered by several authors. Cai and Chen [23] first conducted numerical simulations of the in-plane response of inclined cables to horizontal sinusoidal motion of the upper end. Nayfeh et al. [24] presented theoretical and experimental nonlinear responses of the similar system of a taut string subjected to end excitation at an angle to the cable axis. The method of multiple scales and a numerical continuation technique were used to find steady state solutions and their stability. Resonant responses were found in up to three modes; the directly excited