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# On inertia nonlinearity in irregular-plan isolated structures under seismic excitations

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ABSTRACT

The influence of nonlinear inertia as a function of acceleration, velocity, and displacement is investigated for an asymmetric isolated structure. Six degrees of freedom (6-DOFs) are defined to illustrate translational and rotational displacements of the superstructure and base isolation. Motion equations of such DOFs are derived using the Lagrangian formalism. Two coordinate systems of the reference are defined, one fixed on the building base (global coordinate) and the other at the torsional isolation level (local coordinate). The motion governing equations in the conventional approach is formulated on a linear form in the global coordinate system, whereas in the novel approach, the local coordinate system leads to a nonlinear form of dynamic equations. The difference between two linear and nonlinear models is appeared because of the existence of nonlinear inertia terms just in the nonlinear one. Afterwards, three particular types of isolated structures are employed with the peculiar ratio of torsional–lateral coupled frequency on symmetric frequency. Numerical analysis is applied to investigate the performance of two structural models by exerting harmonic excitations and earthquakes. The results are obtained while analyzing time history and frequency content and show that the coupling effects of nonlinear inertia lead to differences in the responses of linear and nonlinear models of such structures; also, some nonlinear phenomena such as energy transfer between modes, saturation, rigid displacement, and super-harmonic created due to geometrical (inertial) nonlinearities are studied.



## 1. Introduction

Detailed and accurate understanding of the dynamic characteristics of an isolated structure is essential and necessary for the design and control. Although isolators reduce the effects of unfavorable vibrations and movements of the structures and so increase the span of working life, their analysis needs precise attention. Many of the dynamic properties can only be modeled by nonlinear governing equations. On the other hand, there is a wide range of important and risky phenomena which have a nonlinear nature. This nonlinearity can have considerable effects, even if the response domains are very small [1]. However, few research of structural engineering has dealt with the effects of these nonlinearities in isolated structural systems; so, most of them have been modeled by linearization process.

Many of the previous works have emphasized linearization; in other words, the linear model of isolated structure has commonly studied in the global coordinate system. Kilar and Koren [2] determined the most appropriate distribution of

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**Nomenclature**

| | |
|---|---|
| $A$ | excitation amplitude |
| $B$ | base isolation system |
| $C.M_J$ | center of mass |
| $C.R_J$ | center of stiffness |
| $c_{IJ}$ | sum of $I$ direction damping |
| $e_{xJ}$ | eccentricity in $X$ direction |
| $H$ | subscript denotes global coordinates, $X$ or $Y$ |
| $I$ | subscript denotes local coordinates, $x$ or $y$ or $\theta$ |
| $J$ | subscript includes $b$ or $s$, brief of base isolator and superstructure, respectively |
| $k_{IJ}$ | sum of $I$ direction stiffness for $J$'s floor |
| $k_{yiJ}$ | $y$ direction stiffness of the ith resisting element in $J$'s floor |
| $k_{\theta RJ}$ | torsional stiffness about the center of resistance in $J$'s floor |
| $L$ | Lagrangian function |
| $M$ | number of base isolation resisting elements in $Y$ direction |
| $m_J$ | mass of $J$'s floor |
| $N$ | number of base isolation resisting elements in $X$ direction |
| $P$ | subscript includes 1 or 2, denoting the number of $X$ direction's (symmetric) coupled modes. |
| $p$ | number of superstructure resisting elements in $X$ direction |
| $q$ | number of superstructure resisting elements in $Y$ direction |
| $Q$ | subscript includes 1, 2, 3, or 4, denoting the number of lateral–torsional coupled modes. |
| $r_g$ | magnitude of ground excitation |
| $r_J$ | radius of gyration about center of mass of $J$'s floor |
| $S$ | superstructure system |
| $T$ | kinetic energy |
| $u_{gH}$ | $H$ component of $r_g$ |
| $u_{HJ}$ | $H$ direction displacement in global coordinate |
| $u_{IJ}$ | $I$ direction displacement in local coordinate |
| $u_{\theta J}$ | torsional displacement |
| $V$ | potential energy |
| $x_{iJ}$ | distance of the ith resisting element to $Y$ axis |
| $y_{iJ}$ | distance of the ith resisting element to $X$ axis |
| $\Omega$ | dimensionless ratio of excitation frequency to the first symmetric mode |
| $\beta$ | excitation arrival angel from $X$ direction |
| $\theta_J$ | $J$'s floor Rotational angle |
| $\varepsilon$ | non-dimensional bookkeeping parameter |
| $\xi_{IJ}$ | linear damping ratio of $I$ direction for $J$'s floor |
| $\tau$ | non-dimensional time with time scale $\omega_{xs}^{-1}$ |
| $\omega$ | excitation frequency |
| $\omega_{xP}$ | $P$th frequency of symmetric coupled modes |
| $\omega_{IJ}$ | uncoupled frequency of $I$ direction of $J$'s floor |
| $\omega_{y\theta Q}$ | Qth frequency of torsional–lateral coupled modes |

isolators in the asymmetric plan and observed when the center of mass is based on the distribution center of the base isolation, the torsional effect is reduced in the isolation system and more damage is also created in the resistant elements of the flexible side (away from the center of stiffness). De La Llera et al. [3] represented analytical and experimental responses of asymmetric linear structures with frictional and viscoelastic dampers. The results were obtained according to the mass and stiffness of the six-story asymmetric model and the similar responses were generated under both frictional and viscoelastic dampers. De La Llera et al. [4] also studied earthquake response of the isolated structure with lateral–torsional coupling. The basic goal was to provide a simple estimation process of edge displacement of the superstructure and base isolation and compare it with uniform building code (UBC) formula. The results showed that the UBC did not have any accurate estimation of the edge displacement under the static approximation, so it required precise revision in the mentioned subject. Finally, Seguín et al. [5] considered the structure model as two distinct sections: superstructure and base isolation, and represented that torsional balance of the superstructure significantly increased when the base isolation was torsionally flexible or the center of the base isolation and superstructure were placed along the same orientation. Sharma and Jangid [6] utilized the motion equations of the isolated structure and showed that high initial stiffness in the isolation system created intense modes in the superstructure that could lead to more displacement at the story level. The common point of such research was to exploit the dynamics of linear models of asymmetric isolated structures caused by linear lateral–torsional coupling in the global coordinate. Even if an existence of nonlinearities was observed, it was only related to the nonlinear terms of stiffness and damper in the motion governing equations of the isolated structure.

In this way, seismic behavior of single degree of freedom (sdof) isolator with nonlinear damping has been also highly regarded by the researchers [7–10]. The force transmissibility and displacement transmissibility characteristics of vibration isolators have been investigated by forming the governing nonlinear equation of motion. By utilizing the perturbation methods such as multiple scales, harmonic balance [7,9], and output frequency response function (OFRF) [8,10], the nonlinear equations are approximately solved and, then, stability analysis is also carried out due to the first order asymptotically approximation. For this purpose, Ibrahim [11] represented a comprehensive review of nonlinear vibration isolation systems including basic characteristics of nonlinear isolator and their material to the base isolation of structures. Tang and Brennan [7] assumed two different connections of damper and spring to the mass so that damping force was just proportional to the cubic damping. The results showed that both nonlinear systems acted alike while considering force transmissibility; but, the contrariwise occurred when displacement transmissibility was considered. Sun et al. [12] focused on 2-DOFs isolation system with geometric nonlinear damping and observed that the mass of the protected object and the base had a