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Full 3D dispersion curve solutions for guided waves in generally anisotropic media

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ABSTRACT

Dispersion curves of guided waves provide valuable information about the physical and elastic properties of waves propagating within a given waveguide structure. Algorithms to accurately compute these curves are an essential tool for engineers working in non-destructive evaluation and for scientists studying wave phenomena. Dispersion curves are typically computed for low or zero attenuation and presented in two or three dimensional plots. The former do not always provide a clear and complete picture of the dispersion loci and the latter are very difficult to obtain when high values of attenuation are involved and arbitrary anisotropy is considered in single or multi-layered systems. As a consequence, drawing correct and reliable conclusions is a challenging task in the modern applications that often utilize multi-layered anisotropic viscoelastic materials.

These challenges are overcome here by using a spectral collocation method (SCM) to robustly find dispersion curves in the most complicated cases of high attenuation and arbitrary anisotropy. Solutions are then plotted in three-dimensional frequency-complex wavenumber space, thus gaining much deeper insight into the nature of these problems. The cases studied range from classical examples, which validate this approach, to new ones involving materials up to the most general triclinic class for both flat and cylindrical geometry in multi-layered systems. The apparent crossing of modes within the same symmetry family in viscoelastic media is also explained and clarified by the results. Finally, the consequences of the centre of symmetry, present in every crystal class, on the solutions are discussed.

1. Introduction

In the study, and engineering application, of elastic guided waves the dispersion curves of waves propagating within a given structure are a vital tool for inspection purposes and for obtaining information about the properties of the material itself. Dispersion curves for propagating and non-propagating modes are often computed for zero, or low, attenuation values and presented as two or three dimensional plots. See for instance [1–3] or more recently [4–6]. However, cases where high values of attenuation are present and arbitrary anisotropy in single or multi-layered structures is considered have been hardly studied due to the great difficulties encountered when computing their dispersion curves. For instance, dispersion curves for viscoelastic monoclinic plates have been found and, for low values of the attenuation, presented in two

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dimensional plots [7], but in general, a better understanding of the nature of the modes, and a clearer visualization of the solutions, is achieved when dispersion loci are reliably found for low *as well as* high values of attenuation and therefore dispersion curves are required in three dimensions.

In the literature, modes have been categorized in a number of ways and here the classification of Auld [8] is used: propagating modes have real wavenumber, for this reason they are often the most useful for engineering application in NDT since they propagate and transport energy within the structure without attenuation. The second type of solution is non-propagating modes with complex or purely imaginary wavenumber. Note that these modes with complex or purely imaginary wavenumbers are also present in perfectly elastic materials without any energy leakage, those solutions represent local modes that would exist at discontinuities. In addition, non-propagating modes are of interest in certain applications such as the solution of scattering problems using modal summations where a knowledge of the full set of possible modes is essential [9]. A detailed discussion about the physical properties and energy transportation of non-propagating modes in elastic (lossless) media is in the second volume of Auld [8]. For viscoelastic materials, only propagating modes with complex wavenumber exist lest unphysical results are obtained as pointed out in [6]. Propagating modes in viscoelastic media can in turn be split into lowly attenuated modes, with almost real wavenumbers and therefore very useful for inspection purposes in NDT; and highly attenuated modes, with dominant imaginary part for the wavenumbers. The reader must be aware that there does not seem to be a consensus about this nomenclature, see for example [8,10,11]. In this paper we adopt the convention by Auld [8] and we will try to make it clear what kind of modes we are talking about in each case by specifying the nature of the wavenumber. Collectively, these types of modes constitute the complete spectrum for a given problem and their study and visualization is the object of our current investigation.

Several methods have been developed, and successfully deployed, to compute two-dimensional, and less often three-dimensional, dispersion curves for guided waves in flat and cylindrical geometry: finite element (FE), semi-analytical finite element (SAFE) simulations, root-finding routines based on the partial wave approach (Partial Wave Root Finding, PWRF) and lately, spectral methods have become more popular as a powerful alternative, see [4,12] or [13]. More recently, one of these variants known as Spectral Collocation Method (SCM) has been successfully used to model guided waves in elastic and viscoelastic generally anisotropic media, [14] and [15] respectively.

The main contribution and novelty of the present paper is the deployment of a SCM to compute the full three-dimensional spectrum, including highly attenuated and non-propagating purely imaginary modes, for guided wave problems in three cases (Figs. 4a, 5a, 7 and 9) which, to the best of the authors' knowledge, have not been studied before and comprise up to the *most general type of anisotropy*, namely triclinic crystals. Several other well-known cases have also been presented for validation purposes. Regardless of the case under study, this investigation also emphasizes that with the SCM no additional coding effort is needed to obtain the complete three-dimensional spectrum which contrasts with conventional approaches for which the algorithms become significantly more inefficient when searching in a three-dimensional space and which may require substantial modification to obtain robust solutions. The SCM used here was first presented in a previous work by Hernando et al. [15], though there, attention was restricted solely to lowly attenuated modes. The present work goes one step further and, by carefully sorting the eigenvalues provided by the SCM, completes the picture giving the remaining branches of the dispersion loci for the most general anisotropic triclinic class. It is notable that with technological advances there is a need for this full anisotropic generality and for the robust generation of the corresponding three-dimensional dispersion curves presented in this paper for the first time.

Additional contributions of this paper, arising from the undertaken investigations, are the clarifications concerning the apparent crossing of modes within the same symmetry family for viscoelastic media (Section 3) and the discussion of the implications of the centre of symmetry of *all* crystal classes for the solution (Section 5).

The paper is organized as follows. In Section 2 a very brief description of the SCM, and its implementation is given, this is kept to a minimum to avoid repetition and the reader is referred to [12,14,15] and references therein for all the details about the SCM scheme and an extensive discussion of its principal features when used to solve acoustic guided wave problems. Some modifications are required here and these are outlined. In Section 3, classical examples in flat geometry both for elastic and viscoelastic media are presented as a further validation of the SCM approach. The crossing of modes, which is an important detail in practice, is discussed and elucidated with the aid of a new three-dimensional plot for the dispersion curves of a monoclinic material. The full solution for a new multilayer flat case is presented at the end of Section 3. Section 4 treats cylindrical geometry, firstly a classical example is briefly presented for validation and the section closes with a new illustrative multilayer cylindrical system. Section 5 is devoted to the discussion of the results and conclusions.

2. SCM methodology outline

A comprehensive description of the SCM is in [12,14] and, with particular relevance to the present case, in [15]. The main idea behind the SCM is to convert the partial differential equations of the acoustic field into a generalized matrix eigenvalue problem whose eigenvalues are computed thus yielding the full spectrum. We highlight some minor modifications regarding the post-processing of results which must be done in order to visualize the full three-dimensional solution.

The governing equations of motion for a linear elastic anisotropic homogeneous medium are:

$$\nabla_{iK} c_{KL} \nabla^{\mathrm{sym}}_{Lj} u_j = -\rho\omega^2 u_i \tag{1}$$