Journal of Sound and Vibration 363 (2016) 571–583

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Characteristics of steady vibration in a rotating hub–beam system

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ARTICLE INFO

Article history:
Received 13 May 2015
Received in revised form
13 September 2015
Accepted 4 November 2015
Handling editor: L.G. Tham
Available online 28 November 2015

ABSTRACT

A rotating beam features a puzzling character in which its frequencies and modal shapes may vary with the hub's inertia and its rotating speed. To highlight the essential nature behind the vibration phenomena, we analyze the steady vibration of a rotating Euler–Bernoulli beam with a quasi-steady-state stretch. Newton's law is used to derive the equations governing the beam's elastic motion and the hub's rotation. A combination of these equations results in a nonlinear partial differential equation (PDE) that fully reflects the mutual interaction between the two kinds of motion. Via the Fourier series expansion within a finite interval of time, we reduce the PDE into an infinite system of a nonlinear ordinary differential equation (ODE) in spatial domain. We further nondimensionalize the ODE and discretize it via a difference method. The frequencies and modal shapes of a general rotating beam are then determined numerically. For a low-speed beam where the ignorance of geometric stiffening is feasible, the beam's vibration characteristics are solved analytically. We validate our numerical method and the analytical solutions by comparing with either the past experiments or the past numerical findings reported in existing literature. Finally, systematic simulations are performed to demonstrate how the beam's eigenfrequencies vary with the hub's inertia and rotating speed.

1. Introduction

A flexible beam attached to a rotating hub is a good representative of many engineering machines, such as helicopter blades, solar panels of space vehicles, and robotic arms. A puzzling feature of a rotating beam is that its frequencies and modal shapes may vary with the hub's inertia or its rotating speed. The study on this issue has been pursued for quite a long time, and there are numerous experimental and theoretical results that have been published in hundreds of the research papers. See e.g., the review papers [1,2]. In the existing literature, researchers have proposed different approximations characterizing the vibration responses of the system [3,4]. Although some of them are appropriate for the beam under a specific hub's motion, what is missing from this vast literature is a comprehensive theory that can appropriately analyze the beam's vibration under a general motion of the hub.

By confining the hub's motion with a small rotational speed, early studies [5–7] intuitively assumed that the vibration characteristics of the rotating beam are the same as the one of a cantilever beam. This approximation completely neglects the effects from the hub's motion, thus its rationality is questioned by many researchers. Barbieri and Özgüner [8] showed that the ratio of the inertia moments between the hub and the beam is a significant factor to influence the beam's

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http://dx.doi.org/10.1016/j.jsv.2015.11.007
0022-460X/

eigenfrequencies. A similar claim was also given by Bellezza et al. [9] who analyzed the vibration by a different method. Low and Lau [10] conducted the experiments to measure the frequencies of a series of rotational beams with different lengths. The experiments showed that the measured frequencies markedly differ from those of a cantilever beam without rotation. Yigit et al. [11] got the same conclusion by computing a fully coupled nonlinear equations of a hub–beam system. These studies mentioned above clearly indicate that the hub's dynamics cannot be a priori excluded from the vibration analysis.

For a high-speed beam, it has been realized that the coupling effects from the hub's motion should be carefully considered in the beam's model. Under the hub with a prescribed motion, Kane [12] proposed a comprehensive model that involves the effect of geometric stiffening. Bloch [13] analyzed the dynamical stability of a rotating beam to illustrate why the inclusion of the geometric stiffening is necessary. Following Block, Xiao and Chen [14] adopted Hamilton's principle to derive a self-contained nonlinear model coupled with the geometric stiffening, and analyzed the corresponding stability in a more convenient and general way.

Besides the theoretical development in modeling a high-speed beam, its vibration characteristics have also been investigated by many researchers. Under the hub's motion in a uniform rotation, the existing methods in these vibration analysis mainly fall into two categories: the first involves a *space-discretization-first* (SDF) approach that separates the space–time variable of the vibration responses by assumed mode methods [15,16] or finite element techniques [17–21]. The resulted mass and stiffness matrices are then used to perform the vibrational analysis. In the second approach, which is based on a *time-discretization-first* (TDF) technique [22], one works with an assumption that vibration responses are stable and periodic so that the space–time variable can be expanded via periodic functions with respect to time. This makes the infinite systems of PDE be reduced into an infinite system of ODE in a space domain, by which the beam's vibration characteristics can be determined numerically or analytically [23,24]. These two approaches, however, share a basic shortcoming: they exclude the hub's dynamics from the vibration analysis because the hub's motion is prescribed in advance.

Essentially, a prescribed motion of the hub represents a kinematical constraint applied on the beam's dynamics. Although the kinematical constraint may be feasible for some special hub–beam systems, it is not appropriate for a general hub–beam system, especially for the cases where the hub's dynamics is sensitive to the beam's dynamics. Therefore, the vibration analysis should be performed under a beam's model that couples with the hub's dynamics, instead of the hub's kinematics. As this is done, however, the resulted PDE governing the beam's vibration consists of the coupling terms associated with the time-varying rotating velocity of the hub. To analyze the vibration governed by the special PDE, we first select a time sampling window to admit mean treatment for the time-varying angular velocity, then employ a Fourier series expansion within the time interval to reduce the nonlinear PDE into an ODE in a space domain. Based on the ODE, the vibration characteristics of a general rotating beam can be determined either numerically or analytically.

In Section 2, we employ Newton's law to derive the governing equations of the hub–beam system. Section 3 addresses the procedure of the vibration analysis that involves a difference method to numerically determine the vibration characteristics. In Section 4, an analytical solution is proposed for a low-speed beam that allows the geometric stiffening ignorable. In Section 5, we validate our analytical solution against past experiments, and discuss how the hub's inertia and angular velocity affect the beam's eigenfrequencies. The vibration characteristics of a high-speed beam, where the geometric stiffening is necessary in the beam's model, are investigated in Section 6. Conclusions are drawn in Section 7.

2. Equations of a rotating Euler–Bernoulli beam

Fig. 1 shows a hub–beam system: a slender uniform beam with length L and cross-section area A is attached to a hub with a radius R and an inertia J_h about its rotational axis. The density and the elastic modulus of the material of the beam are denoted

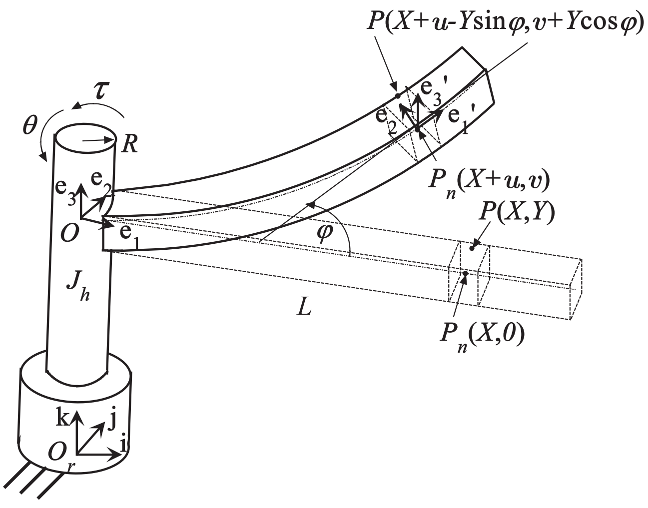

Fig. 1. A flexible beam rotating on a horizontal plane.