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Flow-induced instabilities of shells of revolution with non-zero Gaussian curvatures conveying fluid

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ABSTRACT

We study flow-induced instabilities of axis-symmetric shells of revolution with an arbitrary meridian and non-zero Gaussian curvatures. We consider a fluid–structure interaction (FSI) model based on an inviscid flow model and a thin shell theory. This FSI model is solved using a method that combines the Galerkin technique with the boundary element method (BEM). The present method is capable of investigating the dynamic behavior of doubly-curved shells in contact with flow without the need for an analytical solution of the perturbed flow potential. Shells of revolution with different values of non-zero Gaussian curvatures are investigated and their behavior is compared to shells with zero Gaussian curvature. It is found that the added mass natural frequencies of shells of revolution are larger than those of conical shells with the same inlet, outlet and length. Shells of revolution, with both positive and negative Gaussian curvatures, lose their instability by buckling, however, shells with negative Gaussian curvatures buckle at modes similar to those observed in uniform and conical shells, while shells with positive Gaussian curvatures buckle with localized deformations close to the area with higher local flow velocities.

1. Introduction

The dynamics of shell structures in contact with fluid flow have been studied extensively both experimentally and theoretically because of the applications of such structures in engineering and biomechanics systems. The main focus of the existing studies is on the problem of cylindrical shells with a uniform circular cross-section. These studies have been discussed comprehensively in recent books by Païdoussis and Amabili [1,2]. Recently, there has been an increasing interest in understanding the dynamics of shells with non-uniform cross-sections conveying fluid, with a focus on conical shells.

Thin-walled conical shells have several important applications in submarines and offshore drilling rigs. Kurma and Ganesan [3] used a finite element method (FEM) to study the dynamics of conical shells conveying fluid with various semi-vertex angles. They found that there is a correlation between the shells' circumferential buckling mode and the circumferential mode with the lowest added mass frequency (the natural frequency of the shell filled with fluid). Kerboua et al. [4] used a semi-analytical FEM to study this system. The displacement functions of the structure were derived from the exact Sander's thin shell equation for conical shells, while the flow potential solutions were written in polynomial expansions, based on Frobenius method. Bochkarev and Matveenko [5] studied the dynamic behavior of conical shells

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Nomenclature

α_0	semi-vertex cone angle of a conical shell
χ	boundary conditions assigned in BEM
$\epsilon_{x,0}$, $\epsilon_{\theta,0}$, $\gamma_{x\theta,0}$	strain components in the mid-plane of the shell
ϵ_{xx}, $\epsilon_{\theta\theta}$, $\gamma_{x\theta}$	strain components in an arbitrary position
λ_m	eigenvalues of the axial mode shapes
ω, f	dimensional and dimensionless natural frequencies
ρ, ν, E	density, Poisson's ratio and Young's modulus
ρ_f	fluid's density
φ, φ_a, φ_b	perturbed flow potentials
$\overrightarrow{x_0}$, $\overrightarrow{x}$	position vectors used in BEM
a	the generalized coordinates of the mode shapes
A_x, A_θ	Lamé parameters
k_θ	curvature along the circumferential coordinate
k_x	curvature along the axial coordinate x
l, h, A, a_1, a_2	length, thickness, cross-sectional area, and the inlet and outlet radii of a shell
M, C, K	shell's mass, damping and stiffness matrices
m, n	axial and circumferential wavenumbers of the mode shapes
M_f, C_f, K_f	mass, damping and stiffness matrices due to the flow force
M_x, M_θ, $M_{x\theta}$	resultant moments
N_x, N_θ, $N_{x\theta}$	resultant forces
O_x	center of curvature along the axial coordinate
P	perturbed pressure
q	the generalized coordinates
Q, $\overline{Q}$	dimensional and dimensionless flux
R_θ^0, a_{sor}	parameters defining the geometries of shells of revolution
R_θ	radius of curvature along the circumferential coordinate
R_x	radius of curvature along the axial coordinate
s	curvilinear coordinate along the shell's meridian
S_0, S_1	shell's inlet, outlet and shell's sidewall
U	flow velocity
u, v, w	axial, circumferential and radial displacements
x, r, θ	shell's cylindrical coordinates

conveying fluid using the same inviscid fluid model as in [4] and with different boundary conditions for the perturbed flow potential. They found that conical shells conveying fluid can undergo flutter or buckling instabilities, depending on the shell's semi-vertex angle and boundary conditions. The aeroelasticity problem of conical shells subjected to supersonic flow has also been investigated by several researchers [6–8]. Usually, the linear piston theory for supersonic flow is utilized in the aeroelastic models.

Shells of revolution are extensively used in different systems, such as pressure vessels and rocket nozzles. The existing studies on shells of revolution are mainly based on FEM [9,10] or generalized differential quadrature method [11,12]. While there is a fair amount of literature on dynamics of shells with cylindrical or conical geometry conveying fluid, the studies on shells of revolution with an arbitrary meridian conveying fluid are quite limited. Ventsel et al. [13] combined the Boundary Element Method (BEM) and FEM to investigate the dynamics of shells of revolution filled with fluid. They studied the effect of added mass of the fluid on the natural frequency of the shell of revolution and their vibration modes. Menaa and Lakis [14] studied the supersonic flutter of a spherical shell with a hybrid FEM method and first-order piston theory. They found that by increasing the radius to thickness ratio of a spherical shell, flutter occurs at a higher dynamic pressure.

In this paper, the flow-induced instabilities of shells of revolution with an arbitrary meridian conveying fluid are studied. The main focus of this study is on doubly-curved shells with non-zero Gaussian curvature. The present algorithm combines Galerkin's method with BEM, which is used to determine the induced flow pressure on the shell's inner wall. Because no

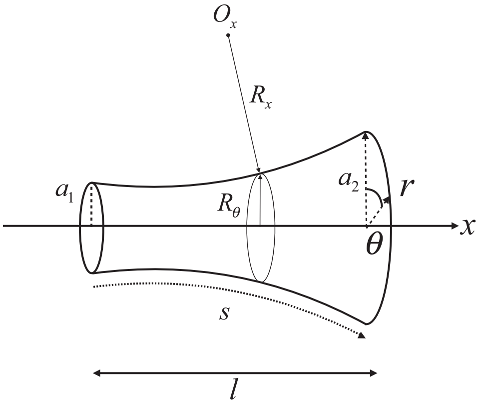

Fig. 1. Schematic of a shell of revolution.