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# An exact generalised function approach to frequency response analysis of beams and plane frames with the inclusion of viscoelastic damping

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ABSTRACT

This paper concerns the frequency response analysis of beams and plane frames with an arbitrary number of Kelvin–Voigt viscoelastic dampers. Typical external and internal dampers are considered, as grounded translational, tuned mass, rotational and axial dampers, for bending and axial vibrations, respectively. Using the theory of generalised functions within a 1D formulation of equations of motion, exact closed-form expressions are derived for beam dynamic Green's functions and frequency response functions under arbitrary polynomial load, for any number of dampers. For a plane frame, exact global frequency response matrix and load vector are built, with size depending only on the number of beam-to-column nodes, for any number of dampers and point/polynomial loads along the frame members. From the nodal displacement solution, the exact frequency response in all frame members is also obtained in closed analytical form. Numerical applications show many of the advantages of the proposed method.



## 1. Introduction

Dampers are very important to control bending and axial vibrations in beams and frame structures [1–3]. Examples are grounded translational dampers (TDs), tuned mass dampers (TMDs), rotational dampers (RDs) and axial dampers (ADs). A typical constitutive model includes linear elasticity and viscous damping, corresponding to a Kelvin–Voigt (KV) model of viscoelasticity [4–11], consistent with Federal Emergency Management Agency (FEMA) code of practise [12]. Also in bolted or welded joints, where flexibility and damping arise due to imperfections or damage, KV RDs [13,14] or ADs [15] are often used.

*Abbreviations:* AD, axial damper; DGF, dynamic Green's function; DSM, dynamic stiffness matrix; EB, Euler–Bernoulli; FE, finite element; FEMA, Federal Emergency Management Agency; FRF, frequency response function; FRM, frequency response matrix; KV, Kelvin–Voigt; LV, load vector; MDOF, multi-degree-of-freedom; MIMO, multi-input–multi-output; RD, rotational damper; SDOF, single-degree-of-freedom; SIMO, single-input–multi-output; SISO, single-input–single-output; TD, translational damper; TM, Timoshenko; TMD, tuned mass damper.

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Within a standard 1D formulation of the vibration problem, several studies have addressed the frequency response of beams with dampers. Frequency response analysis is of great interest as it provides the steady-state response to harmonically varying excitations, as those caused, for instance, by reciprocating or rotating machine parts including motors, fans or compressors. Frequency response data are used for control design, finite element (FE) model updating, system identification or damage detection (e.g., see Ref. [16–18] and the cross references therein). In the context of frequency response analysis, particular attention has been focused on the computation of the dynamic Green's functions (DGFs), which provide the displacement/rotation and stress-resultant responses to a harmonically varying unit point load at an arbitrary position on the beam, based on which the frequency response functions (FRFs) under harmonic distributed loads can be derived by spatial integration over the beam axis. In principle, the DGFs can be built by a classical approach where the steady-state response over every uniform beam segment between two consecutive dampers/point load locations is expressed in a typical trigonometric form with a number of unknown integration constants (4 for the bending problem and 2 for the axial problem), computed by enforcing the B.C. and a set of matching conditions between the responses over adjacent beam segments, at the locations of the dampers and harmonic point load. However, when using this approach the coefficient matrix associated with the equations to be solved has to be re-inverted for any forcing frequency of interest. Of course, it has to be updated whenever the dampers/point load locations change along the axis, and the size of the matrix will inevitably increase with the number of dampers, as expected. Therefore, as beams carrying multiple TDs, TMDs, RDs, ADs are encountered in many engineering applications [4–15], alternative methods to compute the DGFs and FRFs, which may overcome the inherent limitations of the classical approach, have been actively sought in several studies, for both bending and axial vibrations.

As for bending vibrations, the exact DGFs of Euler–Bernoulli (EB) and Timoshenko (TM) beams with KV TDs have been derived by Sorrentino et al. [19–21]. Following a transfer matrix approach in conjunction with an appropriate state-variable representation, the authors obtained the characteristic equation of the free vibration problem as a determinant of a $4 \times 4$ matrix regardless of the number of TDs, and, upon demonstrating orthogonality conditions for the eigenfunctions, they derived exact DGFs by the complex modal superposition method [19,21]. They also built a corresponding exact solution by direct integration method [20]. Exact DGFs by the complex modal superposition method have been obtained for TM beams with KV TDs and RDs by Hong and Kim [22], using the dynamic stiffness matrix (DSM) approach in conjunction with a Laplace transformation of the beam governing equations. In this case, clearly the DGFs are obtained by a matrix whose size increases with the number of dampers. For an EB simply-supported beam carrying a TMD subjected to a harmonic excitation, Tang et al. [23] derived exact DGFs using the recurrence method. Several other authors have shown that approximate, but accurate DGFs of beams with dampers can be derived by a modal representation using the eigenfunctions of the bare beam, i.e. the beam without dampers. Examples may be found in the studies of Wu and Chen [24] for an EB beam with an arbitrary number of TMDs, Gürgöze and Erol [25] for an EB beam with an intermediate viscous TD, an intermediate fixed support and a tip mass. Gürgöze and Erol [26] later applied the method in Ref. [25] to derive the DGFs of a cantilever with an end viscous damper and subjected to external distributed damping. Some further studies have been conducted not only to obtain approximate DGFs, but also to ascertain approximate FRFs under distributed loads. For an EB beam with an arbitrary number of TDs and RDs, Failla [27] built approximate FRFs under arbitrary distributed loads, as superposition of modal FRFs. The modal FRFs have been derived based on appropriate orthogonality conditions for the complex eigenfunctions of the beam with dampers, using the theory of generalised functions to treat the discontinuities of the response variables at the dampers locations. The study in Ref. [27] generalises an approach originally devised by Oliveto et al. [28] for EB beams with end viscous RDs.

In the context of frequency analysis of bending vibrations, it is worth noting that DGFs and FRFs have been sought for EB or TM beams including elastic supports, attached masses or spring-mass systems, but without damping. In particular, exact DGFs have been obtained by the classical approach [29,30] or by using the DGFs of the bare beam in conjunction with appropriate conditions at locations of supports/masses [31–33]. In all these cases, the number of equations to be solved increases with the number of supports/attachments [29–33]. Other exact DGFs have been proposed for EB beams including an arbitrary number of rotational joints modelling cracks [34] by inverting a $8 \times 8$ DSM built by a transfer matrix method. Finally, approximate DGFs have been built as modal superposition of eigenfunctions of the bare beam, for an EB model including a fixed support [35].

As for axial vibrations, the DGFs for a beam with an end viscous damper have been derived in a closed form by Hull [36]. Approximate DGFs and FRFs have been derived by Alati et al. [37] for a beam with an arbitrary number of either TDs/TMDs or ADs, as superposition of modal DGFs or FRFs built upon solving the complex eigenvalue problem and utilising orthogonality of the eigenfunctions. This approach mirrors the one proposed by Failla [27] for the bending problem.

Exact and efficient solutions for frequency response analysis are of great interest not only for single beams but also for frames, as many applications involve frames carrying multiple TMDs, RDs and ADs [4–11,13,14]. The frequency response matrix (FRM) of frames with TMDs has been constructed by Guo and Chen [38] using the reverberation matrix approach in conjunction with generalised matrix inversion. The global DSM, from which the FRM can be derived by matrix inversion [16–18], has been obtained in Ref. [11,13,14] for frames with KV RDs at beam-to-column nodes modelling dissipating beam-to-column connections [11] or bolted/welded joints with imperfections or damage [13,14], and by Caddemi and Caliò [39,40] for frames with elastic rotational joints arbitrarily located along the frame members, using the theory of generalised functions. A recent approach has been devised by Caddemi et al. [41], where a TM beam element with an arbitrary number of deflection and rotation singularities is formulated based on the static shape functions, and corresponding stiffness and