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Vertical vibration of a pile in transversely isotropic multilayered soils

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ABSTRACT

A new method for the dynamic response of a vertically loaded single pile embedded in transversely isotropic multilayered soils is proposed in this paper. The dynamic response of the pile is governed by the one-dimensional (1D) vibration theory, and that of transversely isotropic multilayered soils is achieved by using an analytical layer-element method. Then, with the aid of the displacement compatibility and the contact forces equilibrium along the pile–soil contact surface, the dynamic pile–soil interaction problem is solved efficiently. The presented solution method is proved to be correct and efficient by comparing the obtained results with other existing solutions. Selected numerical results are presented to study the influence of mass density ratio, length–radius ratio, frequency of excitation, soil anisotropy and hard soil stratum on the pile vertical impedance.

1. Introduction

Dynamic analysis of piles and pile groups in soils is a subject of considerable attention due to its extensive application in the fields of dynamic pile testing, pile driving, and pile foundations in machine foundations and high-speed railway bridges. The main purpose of using pile foundation is to reduce the deformations of structures to an acceptable level under external load. Due to the complexity of the dynamic interaction between pile and soils, many researchers have devoted to this research area for a better understanding of the dynamic behavior of pile foundation in recent decades, and various analytical and numerical methods have been proposed to analyze the dynamic behavior of piles or pile groups embedded in different soils, and to evaluate the empirical correlations of less theoretical basis. According to the solutions for the time-harmonic response of single piles or pile groups, these methods mainly involve the finite element methods, the boundary element methods, the boundary element-finite element coupling methods, the rigorous mathematical methods and other simplified methods.

The finite element method has been used by Liu and Novak [1] to achieve the time-harmonic vibration response of a single pile on the basis of dynamic Green's functions and the thin layer method for the soils; with the same method, Cairo and Dente [2] analyzed the vertical harmonic vibration of pile groups combining with the closed-form stiffness matrices derived by Kausel and Roësset [3]. Shi et al. [4] utilized the thin layer element method and the flexible volume method to investigate the pile groups–saturated soil interactions, where the piles are modeled as 3D beam element based on the Euler–Bernoulli beam theory. The boundary element method is generally considered as a powerful numerical method for its

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ability to represent the pile–soil interaction problems and its relatively higher computing efficiency [5,6]. Sen et al. [7] and Mamoon et al. [8] used the boundary element method to investigate the impedance and compliance functions of pile groups embedded in an elastic half-space, and then Maeso et al. [9] studied the harmonic dynamic responses of piles groups in saturated soils. In combination with their respective advantages of the boundary element method and the finite element method, a boundary element-finite element coupling formulation is proposed for the dynamic analysis of piles and pile groups embedded in an elastic half-space [10], elastic layered soils [11] and viscoelastic and poroelastic soils [12]. Since Muki and Sternberg [13,14] proposed the rigorous mathematical model for elastostatic case by decomposing the pile–soil interaction system into a fictitious bar and an extended half-space, some researchers have extended this model to analyze the dynamic response of a single pile embedded in an elastic medium [15–17] or a poroelastic layered half-space [18]. In addition, Wang et al. [19] and Li et al. [20] used this model to evaluate the vertical impedance characteristic of pile groups in a poroelastic medium. In a different way, Rajapakse and Shan [21] introduced the Lagarange′s equation of motion and a discretization scheme to evaluate the axial impedance of a single pile based on Green′s functions for an elastic half-space corresponding to a harmonic ring loads. With the aid of elastodynamic variational principle and Green′s functions for an elastic half-space, Rajapakse [22] presented a more simplified solution scheme to study the dynamic behavior of a long bar in an elastic half-space. A direct formulation proposed by Kaynia and Kausel [23] is carried out to achieve the dynamic stiffness and damping of a single pile and pile groups in a layered half-space combined with Green′s functions for layered media. By coupling the finite element and the consistent infinitesimal finite element cell method, Emani and Maheshwari [24] investigated the influence of pile groups with cap embedded in elastic soils on the dynamic impedance. Instead of the rigorous numerical solutions with a more practical approach, simplified procedures were developed for computing the time-harmonic dynamic response of pile groups embedded in soil deposits [25–29].

Due to the preferred orientation of natural soil in deposition, the mechanical behavior of deposit soils generally appears different in horizontal and vertical direction. From the research results mentioned above, the influence of transversely isotropy of embedding medium on the dynamic impedance of single piles or pile groups is seldom taken into account. Therefore, it is of great significance to determine the influence of transversely isotropic layered medium. In addition, most of the analytical methods mentioned above require significant computational efforts due to a large number of equations for the pile groups with cap and complex compatibility conditions. Thus, presenting a simple and numerically efficient approach is of great significance to the analysis of dynamic response of a single pile or pile groups in transversely isotropic multilayered media and to the application in engineering practice.

This paper aims to propose a practical and computationally convenient scheme for the time-harmonic vertical dynamic response of a pile embedded in transversely isotropic layered soils. This method utilizes the global stiffness matrix for a transversely isotropic multilayered half-space subjected to an interior circular time-harmonic patch load as the fundamental solution, which is derived by Ai et al. [30] through applying the analytical layer-element to each soil layer. By virtue of the advantage of the analytical layer-element only associated with material parameters and negative exponentials, the global stiffness matrix is solved with good numerical stability and low computing times. While the pile is governed by 1D bar vibration theory valid for long bar, and is subdivided into a number of elements which is formulated in a simple way. Then, with a simplification of interaction force acting on each element of the pile, a matrix equation governing the dynamic response of the piles is constructed. By combining with the global stiffness matrix for layered soils and the matrix equation for piles, the pile–soil interaction problem is solved through considering the compatibility of the vertical displacements and the equilibrium of contact forces at pile–soil interface. Finally, the accuracy of the proposed method is verified through comparing the results of this paper with those provided by existing rigorous theory, and some numerical results for vertical impedance are presented to investigate the influence of mass density ratio, length–radius ratio, frequency of excitation, soil anisotropy and hard soil stratum.

2. Solution for axially loaded piles in transversely isotropic multilayered soils

As shown in Fig. 1, an elastic pile of radius a and length h ($h/a \gg 1$) is embedded in a transversely isotropic multilayered half-space and subjected to a prescribed axisymmetric time-harmonic vertical displacement $u_1 e^{i\omega t}$ at its pile head, where u_1 and ω are the amplitude and circular frequency of the motion, respectively, and $i = \sqrt{-1}$. Each soil layer is modeled as a transversely isotropic material of the shear modulus G_{vj}, the vertical Young′s modulus E_{vj}, the horizontal Young′s modulus E_{hj}, the Poisson′s ratios characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallelly and normally to the plane μ_{hj} and μ_{vhj}, respectively, and the density of the medium ρ_j. The material parameters with subscript 'j' are referred to the jth soil layer. In addition, the material properties of the pile are denoted by its Young′s modulus E_p and mass density ρ_p (see Fig. 1). Note that the motion considered in this study is under the steady-state condition, the factor $e^{i\omega t}$ from all expressions is omitted for brevity. It is reasonable to assume that the pile is fully bonded to the embedding soils which could be appropriately modeled using 1D theory for $h/a \gg 1$, whereas the embedding soils are modeled using the 3D continuum model.

With the aid of the 1D governing equations for time-harmonic axial motion of the elastic pile, the following equations can be obtained:

$$q(z) = \frac{dN(z)}{dz} - \rho_p A_p \omega^2 u_p(z) \tag{1}$$