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Non-stationary resonance dynamics of a nonlinear sonic vacuum with grounding supports

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ABSTRACT

In a recent work [L.I. Manevitch, A.F.Vakakis, Nonlinear oscillatory acoustic vacuum, *SIAM Journal of Applied Mathematics* 74(6) (2014), 1742–1762] it was shown that a periodic chain of linearly coupled particles performing low-energy in-plane transverse oscillations behaves as a strongly nonlinear sonic vacuum (with corresponding speed of sound equal to zero). In this work we consider the grounded version of this system by coupling each particle to the ground through lateral springs in order to study the effect of the grounding stiffness on the strongly nonlinear dynamics. In that context we consider the simplest possible such system consisting of two coupled particles and present analytical and numerical studies of the non-stationary planar dynamics. The most significant limiting case corresponding to predominant low energy transversal excitations is considered by taking into account leading order geometric nonlinearities. Then we show that the grounded system behaves as a nonlinear sonic vacuum due to the purely cubic stiffness nonlinearities in the governing equations of motion and the complete absence of any linear stiffness terms. Under certain assumptions the nonlinear normal modes (i.e., the time-periodic nonlinear oscillations) in the configuration space of this system coincide with those of the corresponding linear one, so they obey the same orthogonality relations. Moreover, we analytically find that there are two transitions in the dynamics of this system, with the parameter governing these transitions being the relation between the lateral (grounding) and the interchain stiffnesses. The first transition concerns a bifurcation of one of the nonlinear normal modes (NNMs), whereas the second provides conditions for intense energy transfers and mixing between the NNMs. The drastic effects of these bifurcations on the non-stationary resonant dynamics are discussed. Specifically, the second transition relates to strongly non-stationary dynamics, and signifies the transition from intense energy exchanges between different particles of the system to energy localization on one of the particles. Both of these regimes, as well as transitions between them are adequately described in the frameworks of the new concept of Limiting Phase Trajectories (LPTs) corresponding to maximum possible energy exchanges between the oscillators. Hence, a further example is provided by a particle chain where geometric nonlinearities lead to the realization of a sonic vacuum.

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1. Introduction

It was shown recently [1] that in the limit of low energy a fixed–fixed chain of linearly coupled particles performing in-plane transverse oscillations possesses strongly nonlinear dynamics and acoustics due to geometric nonlinearity, forming a *nonlinear acoustic vacuum*. This designation denotes the fact that the speed of sound as defined in the sense of classical acoustic theory is zero in that medium, so the resulting equations of motion lack any linear stiffness components. An unexpected feature of that system was the presence of strongly *non-local* terms in the governing equations of motion (in the sense that each equation directly involves *all* particle displacements), in-spite of the fact that the physical spring-mass chain has only local (next-neighbor) interactions between particles. These non-local terms constitute a time-dependent 'effective speed of sound' for this medium, which is completely tunable with energy. A rich structure of resonance manifolds of varying dimensions were identified in the nonlinear sonic vacuum, and 1:1 resonance interactions are studied asymptotically to prove the possibility of strong energy exchanges between nonlinear modes.

The concept of nonlinear sonic vacuum is not new, since ordered arrays of spherical elastic granular particles (beads) have been known to exhibit such strongly nonlinear dynamical behavior [2–4]. When no pre-compression exists in this type of ordered granular media so that separations between beads are possible, their nonlinear acoustics are essentially nonlinear, since they lack any linear component, and the corresponding speed of sound is equal to zero – hence, their characterization as 'sonic vacua' by Nesterenko [2]. The feature of sonic vacuum is exclusively related to the Hertzian law during bead–bead interactions, and the basic difference with the systems considered in [1] is that the essential nonlinearity required for realizing the sonic vacuum is smooth and is generated by geometrically nonlinear streatching effects and not by Hertzian interactions or collisions between particles.

Returning to the sonic vauum studied in [1], one of its distinctive features was that its nonlinear normal modes – NNMs [5] could be exactly determined; by NNMs we denote here the time-periodic free oscillations of an undamped dynamical system with straight trajectories in the configuration space. Moreover, the analysis has shown that the number of NNMs in the sonic vacuum was equal to the dimensionality of the configuration space and that no NNM bifurcations were possible. In addition, the most intensive 1:1 resonance intermodal interaction was the one realized by the two NNMs with the highest wavenumbers. Hovewer, the unstretched string model considered in [1] is in some sense a special case, since one of the most significant features of dynamical systems with homogeneous potentials is that the number of NNMs may exceed the number of degrees of freedom due to mode bifurcations [6]. One can expect that such NNM bifurcations will also lead to drastic modification of the non-stationary resonance dynamics of the sonic vacuum, e.g., described by limiting phase trajectories – LPTs [7,8]. LPTs correspond to regimes of most intense energy exchanges between different parts of a dynamical system, and their role in the theory of non-stationary resonant dynamics is similar to the role of NNMs in stationary resonant dynamics. It follows that it is of interest to consider an extension of the nonlinear sonic vacuum developed in [1] so that the modified system has the capacity to undergo NNM bifurcations. Such a study will provide us with the opportunity to investigate how these bifurcations can affect the non-stationary resonant dynamics and corresponding resonant energy exchanges that are realized.

Accordingly, in this work we present an extension of the nonlinear sonic vacuum by considering a structural modification of the particle chain considered in [1] through the addition of a pair of grounding lateral stiffnesses to each particle. Due to geometric nonlinearity generated by the in-plane oscillations of the particles the grounding stiffnesses introduce essentially nonlinear (i.e., nonlinearizable) stiffness terms in the equations of motion of the sonic vacuum and give rise to NNM bifurcations (in contrast to the ungrounded sonic vacuum where no NNM bifurcations are possible). Hence, viewed in another context, the system considered in this work can be regarded as the simplest discrete essentially nonlinear membrane. Due to the complexity of the problem we present analytical and numerical studies of the non-stationary planar dynamics of the model of only two particles under conditions of 1:1 resonance, and analyze the drastic effects of NNM and LPT bifurcations on the non-stationary resonant dynamics of this system.

2. Nonlinear sonic vacuum

We consider the nonlinear chain depicted in Fig. 1. It consists of n particles of identical mass m connected by linear interchain springs of elastic constant k_1; moreover, each particle is connected to the ground by two linear lateral springs of elastic constant k_2. It is assumed that all particles perform in-plane oscillations in the vertical plane (Oxy), and that all springs are unstretched at the system equilibrium corresponding to the line $y=z=0$ (see Fig. 1). In addition, fixed–fixed boundary conditions are assumed for the particle chain, the unstretched length of the ith interchain spring connecting particles $i-1$ and i is being taken equal to l_i, for $i=1,2,\ldots,n$, and the unstretched lengths of the lateral springs are assumed to be equal to d. Considering the free in-plane oscillations of this system, the transverse and axial deformations of particle i are denoted by v_i and u_i, respectively, and the deformed length of the ith interchain spring by l_i' and of the ith lateral springs by d_i' (both lateral springs have equal stretched lengths due to symmetry). Without loss of generality gravity forces are disregarded, and it is assumed that no dissipation forces exist. Finally, without loss of generality the normalization $\Sigma_{i=1}^{n+1} l_i = 1$ is imposed for the interchain springs. Then the analysis follows the approach developed in [1] for the corresponding system with no lateral grounding supports.