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Measurement-based determination of the irrotational part of the structural intensity by means of test functional series expansion

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ABSTRACT

The irrotational part of the structural intensity for flexural wave motion in plate-like structures is calculated from measurement data of kinematic quantities, such as out-of-plane velocities, by a new method that makes use of a test functional series expansion. The computation of the structural intensity and its irrotational part, which allows to effectively assess vibrational sources and sinks, is less sensitive to measurement noise as compared to standard approaches.

1. Introduction

The assessment of structural energy flow is an important diagnostic tool in noise and vibration control. The determination of structural intensity from measurement data of the flexural wave motion in plate-like structures allows the localization and analysis of vibrational sources and sinks. Knowledge about the vibrational sources and sinks can assist professionals to devise appropriate solutions to noise and vibration problems.

Traditionally, structural energy flow is determined by means of accelerometer measurements, in combination with a finite difference scheme (in time domain) to calculate the structural intensity vectors that go along with flexural waves [1]. Verheij proposed to calculate the structural intensity in the frequency domain by means of a cross-spectral analysis [2]. For spatially dense measurements, a spatial Fourier transform is often used [3,4], combined with a laser Doppler measurement technique [4–7].

The structural intensity vector field basically consists of a rotational and an irrotational part. The irrotational intensity indicates how the energy flows from the sources towards the structural "far field". The rotational part indicates how the energy loops, and is not related to the energy flow at large distances from the source. The extraction of the irrotational intensity has significant advantages in that it visualizes the energy flow without the masking effects of energy loops that are related to the rotational intensity. In the literature, however, only a few applications of the irrotational part of the intensity are available [7–9].

This paper introduces a new approach to calculate the irrotational part of the intensity from a spatially dense measurement of kinematic quantities, such as the out-of-plane velocity as measured by means of a laser Doppler vibrometer

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scanning system. Section 2 gives a brief overview of some methods that are commonly used in this field of research, such as the use of a regressive discrete Fourier series, as developed by Arruda [10] (Section 2.1), to regularize the measurement data and a spatial Fourier transform approach (Section 2.2) to determine the irrotational part of the sound intensity. The theory introduced in this paper (Section 2.3) does not use a spatial Fourier transform approach, but uses a series expansion consisting of a limited number of test functions to determine the irrotational part of the sound intensity. This method will be denoted by TFS (test functional series) method. In Section 3 the method is demonstrated for the assessment of energy flows in a vibrating light-weight building element.

2. Theory

Assuming the structural intensity to be a continuous vector field whose divergence and curl vanish at infinity, a Helmholtz decomposition can be applied, decomposing the field in an irrotational (curl-free) part and a rotational (divergence free) part,

$$\vec{I}(x,y) = \vec{I_\phi}(x,y) + \vec{I_\psi}(x,y) \tag{1}$$

where

$$\vec{I_\phi}(x,y) = -\vec{\nabla}\phi(x,y) \tag{2}$$

and

$$\vec{I_\psi}(x,y) = \vec{\nabla} \times \psi(x,y) \tag{3}$$

The structural intensity is composed of 3 components: bending moment energy contribution, shear force energy contribution and twisting or torsion moment energy contribution [1]. All three individual components of the structural intensity can be split up into a rotational and an irrotational part. The test functional series (TFS) expansion approach can be applied to each individual component as well. Without loss of generality, the part of the structural intensity which is related to the bending waves is considered in this paper.

Several methods are available to determine the structural intensity vector field for bending waves in plate-like structures from out-of-plane velocity or acceleration data [1,11,12]. In this paper the formulation of Arruda and Mas [7] is used, which is based upon the classical plate theory [13] for non-interacting propagating waves (absence of acoustic nonlinearity):

$$\vec{I}(x,y) = \sqrt{D\rho h} \quad \Im\left(v(x,y)\vec{\nabla}v^*(x,y)\right) \tag{4}$$

where $D = Eh^3/(12(1-\nu^2))$, E is Young's modulus, ρ is the density, h is the plate thickness, ν is Poisson's ratio, $\Im(\ldots)$ denotes the imaginary part, and $v(x,y)$ is the velocity field as function of x and y.

Since spatial derivatives are required to determine the structural intensity vectors from the velocity field $v(x,y)$, usually regularization is necessary in order to avoid distortions in (high order) spatial derivatives and large wavenumber components, induced by measurement noise and deviations. A number of regularization techniques could be employed for this purpose, among which wavenumber filters [4,14] or similar techniques. Here, the measured velocity field is regularized by means of a smoothing technique called regressive discrete Fourier series (RDFS) (Arruda [10]), which is briefly outlined in Section 2.1.

In Section 2.2, we determine the irrotational part of the structural intensity, $\vec{I}_\phi(x,y)$ by the method of Pascal [8]. In Section 2.3, an alternative approach, which is based on a test functional series, is proposed. It is shown that regularization of the measurement data is not required for the newly introduced approach Section 2.3, as it inherently regularizes the data. Both approaches are compared in Section 3.

2.1. Regularizing the velocity field by means of regressive discrete Fourier series

Basically, in the method developed by Arruda [10], regularization of the measured velocity field is performed by approximating it by a truncated spatial Fourier series. Only the first terms of the spatial Fourier series are retained, thus ignoring the parts of the signal with a high wavenumbers $k = 2\pi/\lambda$, where λ is the structural wavelength. This spatial filtering procedure allows to suppress deviations in the higher spatial derivatives of the field in the calculation of the structural intensity vectors.

Given experimental data $v(x,y)$, spatially sampled at positions $x = m\Delta x$, $m = 0..(M-1)$ and $y = n\Delta y$, $n = 0..(N-1)$, the velocity field can be written as

$$v(x,y) = \sum_{k=-p}^{p} \sum_{l=-q}^{q} X_{kl} W_{\mathcal{M}}^{mk} W_{\mathcal{N}}^{ln} + \epsilon_{mn} \tag{5}$$

where $W_{\mathcal{M}} = e^{i2\pi/\mathcal{M}}$ and $W_{\mathcal{N}} = e^{i2\pi/\mathcal{N}}$. Typically, $\mathcal{M}$ and $\mathcal{N}$ are chosen larger than, respectively, M and N. ϵ_{mn} is a residual to be minimized. The imaginary number is denoted by i. The number of wavenumber terms used in the regressive Fourier