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On the calculation of the complex wavenumber of plane waves in rigid-walled low-Mach-number turbulent pipe flows

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ABSTRACT

A numerical method for calculating the wavenumbers of axisymmetric plane waves in rigid-walled low-Mach-number turbulent flows is proposed, which is based on solving the linearized Navier–Stokes equations with an eddy-viscosity model. In addition, theoretical models for the wavenumbers are reviewed, and the main effects (the viscothermal effects, the mean flow convection and refraction effects, the turbulent absorption, and the moderate compressibility effects) which may influence the sound propagation are discussed. Compared to the theoretical models, the proposed numerical method has the advantage of potentially including more effects in the computed wavenumbers.

The numerical results of the wavenumbers are compared with the reviewed theoretical models, as well as experimental data from the literature. It shows that the proposed numerical method can give satisfactory prediction of both the real part (phase shift) and the imaginary part (attenuation) of the measured wavenumbers, especially when the refraction effects or the turbulent absorption effects become important.

1. Introduction

The prediction of sound wave propagation in ducts is important for numerous industrial applications. This ranges from noise transmission in industrial plants to the prediction of flow-induced pulsations by vortex shedding in confined flows [1,2]. Also for the design of thermoacoustic devices or understanding combustion instabilities an accurate prediction of damping of acoustic waves is needed [3,4]. In this paper we focus on plane waves propagation in low-Mach-number fully developed turbulent pipe flows. Available theoretical models for calculation of the wavenumbers of plane waves are reviewed. A method for computing the wavenumbers numerically is presented, which could potentially predict the mean flow convection effects, the refraction effects, and the turbulent absorption effects on the plane wave propagation.

Compared to the case of wave propagation in flows, the modeling of the wavenumbers for the no flow case is relatively simple and generally more accurate, and a general review of such models is presented by Tijdeman [5]. In the no flow case, the viscothermal absorption within the acoustic boundary layer at the pipe wall is the main factor that influences the sound

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propagation [6, Chapter 10]. For many applications, the pipe can be considered "wide", i.e. the radius of the pipe is much larger than the acoustic boundary layer thickness. A broadly used model for the viscothermal effects in "wide" pipes is the one proposed by Kirchhoff [7], or the extended version provided by Ronneberger [8].

When a flow is present, the Kirchhoff's model is usually adjusted for the convection (Doppler) effects, by dividing it by $1 \pm M$ for the down- and upstream propagating waves, respectively, see Ref. [9,10]. A more theoretically based model to account for the convection effects is provided by Dokumaci [11], which is an extension of the theory of Zwikker and Kosten [12] for axisymmetric wave propagation in circular pipes allowing for the presence of a uniform mean flow. Nevertheless, none of these models can predict the mean flow refraction effects on the sound propagation, which results from the non-uniform mean flow exhibited in the pipe cross-section [13–18]. Analytical and numerical results [13–15,17], most of which are based on the simplification of neglecting the viscothermal absorption, show that such refraction effects increase with the Helmholtz number and the Mach number. In this paper an attempt is made to predict the refraction effects on the sound attenuation for the downstream propagating wave by using a simple model, which was originally proposed by Pridmore-Brown [13] for sound propagation in a channel with absorptive walls, but is here adopted for pipe flows with rigid walls.

In addition to the mean flow effects (convection and refraction), the interaction of sound and turbulence can also result in contribution to sound attenuation [19–21], and experiments [8,20,22,23] show that such attenuation caused by the turbulent absorption becomes the dominant effect compared to the other (viscothermal effects, and the mean flow effects) when the thickness of the acoustic boundary layer is larger than the turbulent viscous sublayer. Different theoretical models for computing the wavenumbers including such turbulent effects have been proposed over the years [19–22,24–28]. The ones proposed by Peters et al. [22] and by Howe [21] are two of the most commonly used.

Recently the present authors proposed a model in Ref. [29] for the turbulent Reynolds stresses acting on the sound waves, which can be easily implemented in the linearized Navier–Stokes equations (LNSE) [30,31], so as to obtain numerical solutions for the wavenumbers. Compared to the theoretical models which are usually obtained by introducing certain assumptions to the LNSE to arrive at a mathematical simplicity, the numerical solutions required less assumptions, so it is possible to obtain more comprehensive results, i.e. the viscothermal effects, the mean flow effects, and the turbulent absorption effects can all be included. However, in the formulation proposed for the LNSE in Ref. [29], the radial momentum equation is absent due to the boundary layer assumption that the acoustic pressure distribution is uniform in the radial direction, and the radial velocity component is negligible [30,32,33]. This assumption becomes less valid when the refraction effects are significant, since the pressure profile is then distorted due to the refraction and hence is not uniform [13,16,17]. In this paper the radial momentum equation is included in the LNSE.

Another effect which may influence the sound propagation is the so-called moderate compressibility effect of the mean flow. This effect is more significant at moderate Mach numbers such as $M = 0.3$, when the stream-wise inhomogeneity of the mean flow may influence the sound propagation [8]. This moderate compressibility effect is briefly introduced in this paper, it is however not incorporated in the LNSE, since the investigation interest in this study is low Mach number flows, where this effect is considered less important than the others.

The paper is organized as follows: in Section 2, basic linear perturbation equations for the sound waves are introduced, and an eigenvalue approach of solving the equations are briefly summarized. In Section 3, different effects on the wave propagation are reviewed, and some theoretical wavenumber models are introduced. These models, together with the numerical solutions to the perturbation equations, are compared in Section 4 with experimental data from the literature. The comparison is to illustrate the refraction effects on the sound attenuation, as well as the turbulent absorption effects on the wave propagation, including both of the attenuation and the phase shift.

2. Formulation of the problem

For a given frequency f which is below the first cut-on frequency of the pipe, the acoustic pressure field $\tilde{p}$ in the pipe can be decomposed into a downstream and an upstream propagating wave, i.e.

$$\tilde{p} = \left(\hat{p}_{+} e^{-\mathrm{i}k_{+}x} + \hat{p}_{-} e^{\mathrm{i}k_{-}x}\right) e^{\mathrm{i}\omega t}, \tag{1}$$

where $+$ and $-$ denote the downstream and upstream propagating waves, respectively, relative to the mean flow (see Fig. 1), $\omega = 2\pi f$ is the angular frequency, and $k_{\pm}$ are the corresponding wavenumbers. In order to accurately describe the wave propagation, the wavenumbers $k_{\pm}$ should be known.

Normally the values of $k_{\pm}$ for a given frequency range depend on the flow conditions, and the geometry of the setup. In this study we assume that the flow in the pipe is axisymmetric and fully developed turbulent flow, so the mean flow (time average of the turbulent flow) is locally parallel, i.e. the mean stream-wise velocity $\overline{u}_x$ is a function of the radial direction, r, only. In addition, the cross-section of the pipe is assumed to be circular and uniform along the stream-wise direction. These two assumptions allow to represent the plane waves in the Fourier domain with spatially constant wavenumbers, so the waves can be treated locally.

In order to review the models for calculating $k_{\pm}$, the basic linear perturbation equations for the sound waves are first introduced in this section as a foundation of the modeling. Then an eigenvalue approach of solving the perturbation equations is briefly summarized, which provides a numerical manner of obtaining the wavenumbers.