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Damping by parametric excitation in a set of reduced-order cracked rotor systems

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ABSTRACT

A common tool utilized for the stability analysis of parametrically excited linear systems, such as rotors with cracked shafts, is Floquet's method. The disadvantage is a long calculation time needed to evaluate the monodromy matrix and instability zones. An efficient alternative is the generalized Bolotin's method, where the instability zones are evaluated quickly, yet the matrices that must be calculated are of large dimensions.

In the present paper, the stability analysis is conducted with both Floquet's method and the generalized Bolotin's method. However, the order of the model is reduced to two modes only and stability analyses are performed for the second-order systems obtained with various combinations of the reducing modes. Then, the results of such analyses are collected in an overall stability map. The stability map obtained in this way closely reconstructs the stability map calculated with the full-order model of the rotor, yet the calculation time needed to generate the collected map as well as the dimension of the problem are considerably reduced.

The approach is demonstrated with a mathematical model of the machine with the breathing crack modeled using the rigid finite element method. The rotor is not rotating, yet the stiffness of the shaft is varied periodically to simulate the parametric excitation.

An interesting indication of the developing shaft crack observed in the generated stability maps is the presence of anti-resonant zones, where the rotor vibration amplitudes quickly decay. It is anticipated that this phenomenon of increased damping at specific excitation frequencies may have potential application for shaft crack detection.

1. Introduction

Over the years, several methods have been developed and utilized for the stability analysis of rotors with cracked shafts. The small parameter method and the method of averaging were used by Ballo [1] to study flexural vibrations of a continuous slender shaft with a breathing crack. Dai and Chen [2] identified instability zones of a nonlinear cracked rotor with unsymmetrical viscoelastic supports by using the harmonic balance method. The harmonic balance method was also utilized by Sinou [3,4], Didier et al. [5], and Sawicki et al. [6]. Plaut [7], Hegazy et al. [8] and Kamel and Bauomy [9] used the multiple scales method to calculate instability zones of cracked rotors.

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A very popular approach is based on Floquet's theory and has been utilized in several papers, including the most recent by Guo et al. [10] and Ricci and Pennacchi [11]. Earlier, Floquet's method was used by Huang et al. [12], Gasch [13], Meng and Gasch [14], Fu and Zheng [15], and Dai and Chen [2] to calculate stability maps of cracked rotors.

A common problem when utilizing Floquet's method is the calculation of the monodromy matrix, especially for high-order systems. The simple approach to obtain the monodromy matrix by direct integration of the equations of motion over one oscillation period is not time efficient. Therefore, modification of the original Floquet's method has been proposed by Hsu [16], and extended by Huang et al. [17] to increase calculation efficiency [14,18]. Although the calculation time is decreased, the accuracy of calculations is also reduced, which has been discussed in detail by Ricci and Pennacchi [11].

An interesting modification of Floquet's method was proposed by Bolotin [19] for the quick evaluation of the boundaries of instability zones instead of determination of the full stability map. Bolotin's method has been used for years for various stability problems. Han and Chu, adopted this method to analyze the stability of the rotating shaft with two transverse cracks [20], as well as the rotating shaft with an elliptical crack [21]. Turhan [22] suggested an important modification of Bolotin's method, including not only the harmonic and subharmonic but also combination resonances. However, application of the generalized Bolotin's method developed by Turhan requires the evaluation of large dimension matrices. Therefore, Turhan's modification has been only used for low-order systems [23,24].

In the present paper, a new approach to the cracked rotors stability analysis is suggested. Applying the modal transformation, a full-order model of the parametrically excited rotor with a cracked shaft is reduced to the second-order system using only the two first modes of the lowest natural frequencies. The instability zones and the boundaries of these zones for the reduced-order system are evaluated based on Floquet's method and on the generalized Bolotin's method. The procedure repeats for other combinations of the lowest modes of the system. Using this approach, a set of stability maps is obtained for different combinations of the two natural modes, reducing the original high-order system. Next, these stability maps are collected into the overall stability map. The results of numerical experiments show that the resulting overall stability map closely reconstructs the stability map obtained with the full-order model of the rotor. Furthermore, the calculation time needed to evaluate the collected stability map is significantly reduced, as the calculations are performed for a limited set of the second-order systems.

Different methods for order reduction of high-order parametrically excited linear systems to analyze their stability were studied by Pumhössel et al. [25], Sinha et al. [26,27], Villa et al. [28], Sawicki and Gawronski [29]. In the present paper, the simple modal transformation method is chosen as the stability analysis is conducted for subsequent natural modes of the uncracked rotor to find the resulting principal and combination resonances of the parametrically excited rotor with a shaft crack.

A *parametric excitation frequency* η_n is defined as [30–39]

$$\eta_n = \frac{|\Omega_k \mp \Omega_l|}{n}, \quad k, l = 1, 2, \dots \tag{1}$$

where Ω_k and Ω_l are the kth and lth natural frequency of the undamped system with constant coefficients, and n is the order of the parametric resonance. For $k = l$, the frequency η_n is called *a principal parametric resonance,* and for $k \neq l$ – it is a *parametric combination resonance*. Usually, only first-order resonances (for $n = 1$) are included.

It has been shown [31,40] that, principal parametric resonances (for $k = l$ and the plus "+" sign in Eq. (1)) and parametric combination resonances of the summation-type (for $k \neq l$ and the plus "+" sign) always destabilize the system. However, difference-type combination resonances (for $k \neq l$ and the minus "–" sign) for parametric excitations leading to symmetric system matrices have a unique property to suppress vibrations. The specific difference-type combination excitations that stabilize an otherwise unstable system are called *parametric anti-resonances*.

The anti-resonant zones were first observed by Tondl [40], who demonstrated that an unstable self-excited system can be stabilized by introducing periodic stiffness changes at a specific parametric anti-resonance frequency. Since then, parametric anti-resonances have been observed and studied only in combination with self-excitation. Dohnal [30–36] showed that properly chosen parametric anti-resonance not only stabilizes an already unstable system, but it can also increase the existing damping in a stable system. He performed experimental tests using electromagnetic actuators for a two mass system, a cantilever beam, and a flexible rotor, confirming that when excited close to a non-resonant parametric combination resonance a vibrating system enhances its damping properties [35]. Thus, the phenomenon known as the *damping by parametric excitation* has been identified. This stabilizing effect is interesting for its probable ability to indicate a shaft crack.

The damping by parametric excitation in parametrically excited rotors has been already studied by several authors. Dohnal [34,35] conducted numerical and experimental studies of a flexible rotor supported by active magnetic bearings (AMBs) confirming the increase in damping by stiffness changes at the bearings. Ecker [37,38] presented numerical results for a Jeffcott rotor with time-varying stiffness and demonstrated the increase in damping as a result of axial excitations at the first anti-resonant frequency. Similar results were obtained by Ecker [37] for a rotor with periodically varied radial stiffness applied in different locations along the shaft axis.

The problems of modeling the rotors with cracked shafts were studied by several authors. Gasch [41] introduced the hinge model where the stiffness of the cracked shaft changed in a step-like manner, reflecting the open/close states of the crack during the rotation. Mayes and Davies [42] proposed a cosine steering function controlling the opening and closing of the crack. To better describe crack breathing, the Fourier expansion of the steering function was introduced in some papers, e.g. in [6,14]. Sinou and Lees [3] and Al-Shudeifat and Butcher [43] extended and improved the approximate method of estimating the reduction of a second moment of area required to model a crack introduced by Mayes and Davies [42].