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Three-dimensional perturbation solution of the natural vibrations of piezoelectric rectangular plates

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ABSTRACT

The paper discusses a perturbation solution of the natural frequencies and mode shapes of a piezoelectric rectangular plate modelled as a three-dimensional body. The coupled theory of piezoelectricity is used, with the governing equations consisting of one electrostatic and three mechanical equations coupled through the piezoelectric effect. Analytical perturbation formulas up to the first-order terms have been derived and used. An important difference of the present analysis as compared to the classical perturbation method consists in that the small parameter enters not only the governing equations but the boundary conditions as well. To address this complication an efficient new approach that makes use of generalized functions has been proposed. Results of the natural frequencies and mode shapes obtained by the perturbation method are discussed for a thin piezoelectric rectangular plate, a thick plate and a piezoelectric parallelepiped. All the results obtained using the perturbation method have been compared with the exact solutions of the coupled electromechanical problem. The proposed perturbation approach furnishes an efficient approximate method of studying the coupled piezoelectric vibration problem. The main advantage of the method derives from the fact that only the elastic solution is required, the effect of piezoelectric coupling being accounted for at a post-processing stage.

1. Introduction

The analysis of the vibrations of elements made of piezoelectric materials is important in various applications. Classically, piezoelectric materials have been used, e.g., as piezoelectric resonators made of crystals such as quartz and used as elements of electronic circuits, or as ultrasonic sensors and actuators. Nowadays, important new applications have emerged that include smart structures, structural health monitoring and energy harvesting.

The rigorous formulation of piezoelectric problems makes use of the coupled theory of piezoelectricity, which has been discussed, e.g., in Refs. [1–3]. In this theory the problem is described by a set of coupled electromechanical equations and the corresponding mechanical and electrical boundary conditions. Among other results, coupled piezoelectric theory has been applied to the analysis of piezoelectric plates and shells. Approximate two-dimensional theories of single- and multi-layered piezoelectric plates have been discussed, e.g., in Refs. [3–7]. Exact three-dimensional equations have been applied to the analysis of the free vibrations of piezoelectric plates in Refs. [8,9]. The wave propagation in piezoelectric plates with infinite

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in-plane dimensions has been discussed, e.g., in Refs. [1,2,10]. The finite-element formulation of coupled piezoelectricity problems can be found in Refs. [11,12].

The solutions of coupled piezoelectricity problems are obtained either analytically or numerically by solving the coupled electromechanical equations. However, since the effect of piezoelectric coupling on the natural frequencies is moderate, a perturbation approach can be considered as an interesting alternative. The present paper will discuss such an alternative approach to the calculation of the natural frequencies and mode shapes, the latter defined in terms of the displacements and electrostatic potential, using a consistent perturbation approach.

It is worthwhile to point out that the importance of the calculation of the modal parameters of classical elastic structures is due not only to the study of free vibration problems, but even more so to the fact that the modal data can be used in predicting the forced response of linear elastic structures by the mode-superposition method. In order to use the modal data to calculate the forced response, use is made of the fact that the modes of elastic structures satisfy the orthogonality conditions. In this context it is important to observe that orthogonality conditions of natural vibration modes also hold for piezoelectric continua described by coupled piezoelectricity theory [1,2]. According to the discussion in Ref. [2] “the existence of these orthogonality relations allows the forced oscillation problem for a (piezoelectric) resonator to be solved by expanding the fields in a series of modes” (compare Ref. [2], vol. 2, p. 250 and the following, where more information and examples can be found).

A rigorous perturbation approach to solving piezoelectric problems has not been given much attention in the literature. In Ref. [13] the classical perturbation method has been applied to the matrix equation obtained by finite element discretization of an array of piezoelectric transducers. A related sequential approach to the solution of the finite-element matrix equations of piezoelectric continua has been described in Refs. [14,15]. An analytical perturbation approach to the vibration of a piezoelectric plate in cylindrical bending, different from the present analysis, has been presented in Ref. [16].

A perturbation method of calculating the natural frequencies and modes similar to the present paper has been proposed for the first time by the author in Ref. [17], for a simpler case of the through-thickness vibration of a piezoelectric plate with infinite in-plane dimensions. It has been pointed out in Ref. [17] that an important complicating factor in the perturbation solution follows from the fact that the small parameter that describes piezoelectric coupling appears not only in the governing equation, but it also enters the boundary conditions. The method of handling this complication introduced in Ref. [17], in which new functions that describe the displacements and electrostatic potential are introduced in such a way that the boundary conditions become homogenous when expressed in terms of these functions and the equations are modified appropriately by additional terms, has been found to be cumbersome in the present more complex case (even though results have been obtained by this method for plates with electrically short-circuit faces in perfect agreement with the results in this paper). A more efficient approach to handle non-uniform boundary conditions that makes use of generalized functions (distributions) will be introduced in the present paper. To the author's best knowledge, this method of solving coupled piezoelectric problems is new and it has not been discussed before.

In order to verify the accuracy of the method numerical results for the natural frequencies and mode shapes of a thin and thick piezoelectric plate, as well as a parallelepiped, will be discussed, and the perturbation solutions will be compared with the exact ones. Since the detailed knowledge of the modal data of elastic structures (including higher modes) is used in the perturbation analysis, a number of such results for anisotropic elastic plates and a parallelepiped are included in the paper which are of independent interest, apart from the new piezoelectric results.

2. Governing equations of piezoelectric plate vibration

The piezoelectric plate considered in this paper and its dimensions are shown in Fig. 1. The governing equations and the corresponding boundary conditions of the plate made of a ceramic belonging to material symmetry class 6 mm, polarized in the z-direction, have been discussed in full detail in Ref. [8]. Introducing the following non-dimensional quantities:

$$\overline{x}=\frac{x}{a},\quad \overline{y}=\frac{y}{b},\quad \overline{z}=\frac{z}{h},\quad \overline{U}_x=\frac{u_x}{h},\quad \overline{U}_y=\frac{u_y}{h},\quad \overline{U}_z=\frac{u_z}{h},\quad r=\frac{a}{b},\quad \xi=\frac{a}{h},$$

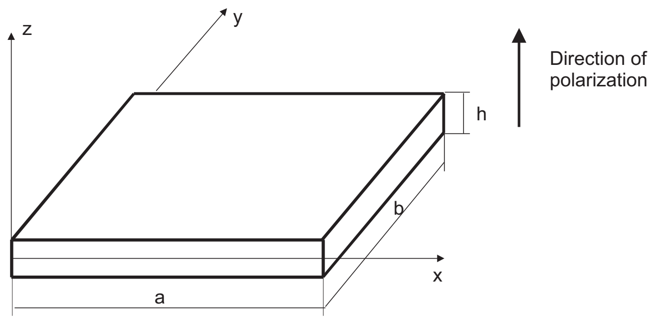

Fig. 1. The piezoelectric rectangular plate.

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