ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Thick hollow cylindrical waveguides: A theoretical, numerical and experimental study

Aleksandra Ziaja [a,b], Li Cheng [a,b,*], Zhongqing Su [a,b], Pawel Packo [c], Lukasz Pieczonka [c], Tadeusz Uhl [c], Wieslaw Staszewski [c]

[a] Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
[b] The Hong Kong Polytechnic University Shenzhen Research Institute, Shenzhen 518057, China
[c] Department of Robotics and Mechatronics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Krakow, Poland

ARTICLE INFO

Article history:
Received 10 October 2014
Received in revised form
26 February 2015
Accepted 2 April 2015
Handling Editor: H. Ouyang

ABSTRACT

The paper investigates elastic waves guided by thick-walled, hollow cylindrical structures. Theoretical, numerical and experimental investigations are presented to facilitate understanding of various wave propagation phenomena in thick-walled cylinders for potential damage detection applications. Semi-analytical analysis of dispersion characteristics is performed, revealing a repetitive pattern of coupled pairs of higher-order longitudinal modes. This behaviour is found to be analogue to terrace-like structures formed by interlacing high-frequency symmetric and antisymmetric plate mode curves. A hyperbolic behaviour of curves and modeshape transitions is observed due to mode coupling. The work presented demonstrates analytically how solutions for a thick-walled hollow cylinder correspond to the Lamb wave theory. The relevant pseudo-symmetry relations for mode displacement patterns are obtained using asymptotic approximations of Bessel functions. Theoretical solutions of dispersion characteristics are compared with numerical simulations that are based on the local interaction simulation approach. The results are validated experimentally using laser vibrometry.

1. Introduction

Damage detection methods based on guided ultrasonic waves have been extensively explored for a last few two decades in the field of Non-Destructive Testing (NDT) and Structural Health Monitoring (SHM). This includes methods that utilise Lamb waves propagating in thin plates and Rayleigh waves confined to surfaces of elastic solids [1–3]. Some of these methods are considered for aerospace and civil engineering applications [4–6]. Therefore research effort related to the understanding of guided wave propagation in complex engineering structures is important. In general, any type of geometry in which waves are bounded by one or more surfaces can be regarded as a waveguide. Plates [7] and hollow cylinders [8,9] are examples of waveguides that have attracted considerable attention.

An analytical solution to the problem of elastic waves in an infinitely long, hollow cylinder was firstly introduced in the late 1950s [10]. That initial work included theoretical and numerical analyses of longitudinal, torsional and flexural modes

* Corresponding author at: Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong. Tel.: +852 2766 6769.
E-mail addresses: aleksandra.ziaja@connect.polyu.hk (A. Ziaja), li.cheng@polyu.edu.hk (L. Cheng), zhongqing.su@polyu.edu.hk (Z. Su), pawel.packo@agh.edu.pl (P. Packo), lukasz.pieczonka@agh.edu.pl (L. Pieczonka), tuhl@agh.edu.pl (T. Uhl), staszews@agh.edu.pl (W. Staszewski).

http://dx.doi.org/10.1016/j.jsv.2015.04.004
0022-460X/

Please cite this article as: A. Ziaja, et al., Thick hollow cylindrical waveguides: A theoretical, numerical and experimental study, *Journal of Sound and Vibration* (2015), http://dx.doi.org/10.1016/j.jsv.2015.04.004

propagating in the axial direction. Further research work in this area includes studies on ultrasonic waves in hollow cylinders reported in [11,12]. However, research on cylindrical guided waves gained a new impetus when real applications (e.g. related to long distance pipe inspection) were established [8,13,14]. Among various analytical methods developed, the Normal Mode Expansion (NME) technique – introduced in [15] – has made a significant contribution to the analyses of non-axisymmetric cylindrical waves. The NME approach yields the solution to wave propagation problem for a given surface loading. This allows one to determine angular profiles of a pipe at any distance simply as a superposition of excited modes with specific amplitudes [14,16]. From damage detection view point, the physical understanding of various wave modes in monitored structures is of paramount importance as the first step in the development of a new damage detection techniques. It appears that – in contrast to thin-walled hollow cylinders for which research studies have been vibrantly evolving (e.g. research work on crack detection using torsional modes [17], nonlinear wave propagation features [18] or Lamb wave tomography [19]) – very little research work has been conducted for thick-walled cylindrical structures (where the wavelengths suitable for crack detection are much less than the wall thickness). To our knowledge, the work reported in [20] is probably the only literature example, which discusses possible applications of ultrasonic guided waves for crack detection in a thick-walled hollow cylindrical structure: a train axle. However, research developments presented in that work neglect other than Rayleigh waves and thus restrict possible applications only to surface crack detection.

The work presented in the current paper attempts to give a broader theoretical understanding of guided wave propagation phenomena in thick-walled cylindrical structures. Various wave propagation features specific to thick-walled cylindrical structures – such as mode interlacing, pseudo-symmetry of modes and modeshape transitions – are explored and investigated. These phenomena are discussed in relation to the well-established theories. The inner radius of the hollow cylinder is considered as an important parameter that relates obtained solutions to these theories; where in the limits – i.e. for nearly zero and infinite values of the inner radius – the wave propagation theory in plates and solid rods applies, respectively. A paradigm relating wave propagation in hollow cylinders and plates is established analytically using an asymptotic approximation of Bessel functions. Differences, similarities and possible discrepancies in dispersion characteristics and particle displacement patterns are investigated via analytical and numerical studies. The influence of structural geometrical variations on both, i.e. axisymmetric and non-axisymmetric, types of modes is also addressed. Semi-analytical solutions are compared with numerical simulations. The latter is performed using the Local Iteration Simulation Approach (LISA). The results are validated experimentally with the help of laser vibrometry.

The paper is organised as follows. Section 2 starts with the theoretical background, followed by semi-analytical analysis of the paradigm that relates wave propagation in hollow cylinders to wave propagation in plates. Then various characteristic features of axisymmetric and non-axisymmetric modes are discussed using dispersion characteristics. Section 3 presents the results of dispersion characteristics evaluated from numerical simulations. The results are compared with experimental measurements in Section 4. Finally, the work is concluded in Section 5.

2. Guided waves in hollow cylinders – theoretical analysis

Typically, the case of low thickness-to-radius ratio of hollow cylinders is considered in the literature as equivalent to the case of thin-walled pipes. Therefore plate theory approximations are frequently employed when wave propagation is investigated. A more general analysis – that also includes thick-walled structures – is presented in this section. The correspondence between wave propagation in plate and hollow cylinders is demonstrated through semi-analytical analysis. In addition, dispersion characteristics and particle displacements are investigated to reveal mode coupling phenomena.

2.1. Theoretical background

For the clarity of further discussions, a brief introduction to basic notations and expressions used for evaluation of dispersion characteristics is given below. The theoretical derivation presented in this section mainly originates from [10]. For more detailed description readers are referred to [21,22].

For an isotropic hollow cylinder, the wave equation can be derived from the Navier equation given as

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

where $\mathbf{u}$ is the displacement vector, λ,μ denote Lamé's constants, ρ is the density and ∇^2 is the three-dimensional Laplace operator. When the Helmholtz decomposition is applied, the displacement vector $\mathbf{u}$ can be decomposed into a ϕ-dilatational scalar potential and a $\mathbf{H}$-equivoluminal vector potential, leading to the wave equations

$$v_c^2\nabla^2\phi=\frac{\partial^2\phi}{\partial t^2}\quad\text{and}\quad v_s^2\nabla^2\mathbf{H}=\frac{\partial^2\mathbf{H}}{\partial t^2}, \tag{2}$$

where $v_c=\sqrt{(\lambda+2\mu)/\rho}$ and $v_s=\sqrt{\mu/\rho}$ are the compressional and shear bulk wave velocities, respectively. Assuming waves propagating in positive axial direction of the cylinder (Fig. 1) and denoting the propagation term by $e^{i(\xi z-\omega t)}$ – where ξ and ω are the wavenumber and angular frequency – solutions in the form of radius r and angle θ dependent functions are sought as

$$\phi=f(r)\cos n\theta\, e^{i(\xi z-\omega t)}$$