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Analysis of resonant pull-in of micro-electromechanical oscillators

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ABSTRACT

In this paper, the equations governing the pull-in of electrostatic (micro-electromechanical systems MEMS) oscillators are established and analyzed. This phenomenon defines the maximal oscillation amplitude that can be obtained without incurring instability and, hence, an upper limit to the performance of a given device. The proposed approach makes it possible to accurately predict pull-in behavior from the purely resonant case, in which the electrostatic bias is very small, to the static case. The method is first exposed in the case of a parallel-plate resonator and the influence of the excitation waveform on the resonant pull-in characteristics is assessed. It is then extended to the more complex case of clamped–clamped and cantilever beams. The results are validated by comparison with transient simulations.

1. Introduction

The static pull-in of capacitive MEMS structures is a well-studied phenomenon, theoretically and experimentally [1,2]. In particular, it is well-known that a voltage-controlled parallel-plate capacitor may not be displaced by electrostatic forces to a position exceeding one third of the gap without incurring instability. The validity of this result is limited to the case when the bias voltage applied to the capacitor is increased quasi-statically or when the structure is critically damped. Several works have been concerned with establishing similar results to the case when the structure is under-damped, mostly when the voltage is applied as a step [3–5] or when it is a superposition of a DC and of an AC component [6–8]. The latter case is of great importance for resonant sensing applications, in which the resonance frequency of a structure is measured to track changes in a physical quantity. In this context, it is important to determine the largest stable oscillation amplitude since it determines the maximal magnitude of the detected signal, and, consequently, sets the constraints on the design of the associated electronics of the MEMS/electronics interconnection scheme [9,10].

The measurement of the resonance frequency of a MEMS structure can be performed using open-loop or closed-loop techniques (Fig. 1). This paper is dedicated to the latter case, in which feedback electronics sense the position of the resonator and actuate it close to its resonance frequency by enforcing a set of so-called “self-oscillation” conditions (Barkhausen criterion) [11]. Although the electrostatic closed-loop scheme is ubiquitous [12–17], its stability has received little attention so far. Most papers dedicated to resonant pull-in are concerned with open-loop techniques [6–8], in which the excitation frequency is controlled and the amplitude and phase of the displacement are free to vary. Only a handful of papers [18–20] are dedicated to resonant pull-in in closed-loop, in which the phase is set and the amplitude and frequency vary. Of these, [18] is dedicated to the case when the resonator is a parallel-plate capacitor actuated with a square-wave voltage and is only valid when the DC bias applied to the structure is very small compared to the static pull-in voltage. These three limitations (geometry, waveform and bias voltage) prevent using the results from [18] in the majority of resonant

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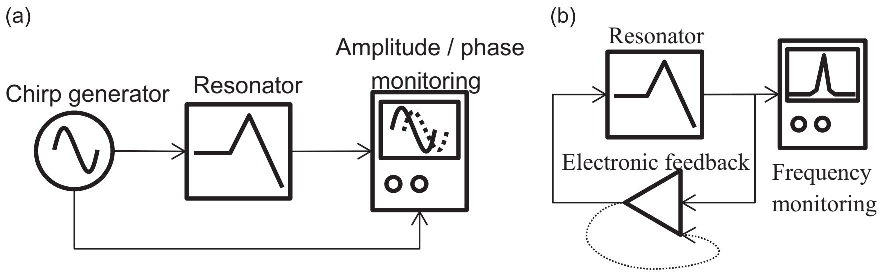

Fig. 1. Open-loop (a) and closed-loop (b) techniques for resonance frequency measurements. In the open-loop technique, the resonator is typically excited with a chirp. The amplitude and phase response of the system are obtained by demodulating the excitation and the resonator output. In the closed-loop technique, the resonator is excited with the signal output by an electronic circuit enforcing some self-oscillation conditions in the loop. The feedback electronics may include a phase-locked loop in order to guarantee a specific input–output phase relationship, as symbolized by the dotted arrow, or may have a purely feed-forward architecture.

MEMS/NEMS applications. Such applications are most often based on sine-wave actuation, operate at finite bias voltage and have flexible moving parts, very far from the plane-capacitance hypothesis. In [19], the framework of [18] is expanded to account for other nonlinearities (caused by strain-stiffening) and the study is made for large DC bias. However, as shown in the present paper, the approach used by the authors of [19] does not take into account the interdependence of the static component of the displacement and the oscillation amplitude. Consequently, this approach leads to qualitatively and quantitatively inaccurate results, *especially at large DC bias*. In [20], resonators of various shapes (parallel-plate, bridge or cantilever beams) with different types of actuation waveforms (square-wave, sine-wave, pulses) and large DC bias are considered. However, the study is limited to the case of symmetric double-sided actuation, in which the static component of the displacement is equal to zero. In an electrostatic MEMS resonator, asymmetry may result from several causes, the most common being that a single electrode is used to actuate the resonator and sense its position; in the present paper, we show that, in such asymmetric cases, the static component of the displacement (resulting from the asymmetry) plays a key role in the pull-in phenomenon.

In this paper, the case of a resonator (parallel-plate, bridge, or cantilever) actuated *from only one side* is considered and the influence of the excitation waveform on the resonant pull-in amplitude is studied. The main contributions of the paper are that

- a general methodology, based on harmonic balancing, for deriving the resonant pull-in condition in asymmetric electrostatic resonators is derived,
- this theory is valid from zero DC bias (purely resonant pull-in, as in [18]) to large DC bias (static, non-resonant pull-in),
- the resonant pull-in conditions are given in terms of oscillation amplitude, oscillation frequency and static component of displacement for common resonator geometries and excitation waveforms,
- voltage pull-in conditions are also established for these geometries and waveforms, in terms of maximal actuation voltage versus bias voltage, and
- it is shown that, as in the case of double-sided actuation [20], the greater travel range for a given DC bias is obtained with pulse-actuation, regardless of the geometry of the resonator.

The outline of the paper is the following. In Section 2, the framework and the model governing the transient dynamics of a parallel-plate resonator are described. In Section 3, the equations governing the averaged dynamics of the model are established, based on a describing function approach [11]. Section 4 presents the method for deriving the resonant pull-in characteristics from the averaged dynamics. The analysis is based on an extension of one of the stability criteria presented in [11] to the case when the displacement of the structure has an amplitude-dependent DC component. This is illustrated in the case of square-wave actuation and shows that the method proposed in [19] for determining resonant pull-in is faulty in the case of a finite DC bias. Section 5 discusses the limits of this work and its extension to more complex cases, in particular to the case of clamped–clamped and cantilever beams. Section 6 contains some concluding remarks.

2. Framework

We consider the one-sided closed-loop electrostatic actuation of a parallel-plate resonator with stiffness K, mass M, natural pulsation $\omega_0=\sqrt{K/M}$ and quality factor Q. An electrode with surface area S_e is situated across a gap G_0 from the resonator, supposed to be very small with respect to the lateral dimensions of the electrode. A voltage $V(t)=V_b(1+v(t))$ is applied across the gap and exerting an electrostatic force on the resonator and setting it into motion. Throughout the paper, the actuation voltage is assumed to be very small with respect to the bias voltage (i.e. $v\ll 1$). The influence of this hypothesis is illustrated in Section 4.3.

The position of the resonator as it moves across the gap is given by $G(t)=G_0a(t)$, so that $a=1$ means that the structure touches the opposite electrode (Fig. 2).

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