Journal of Sound and Vibration 348 (2015) 224–238

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Identification of backbone curves of nonlinear systems from resonance decay responses

Julián M. Londoño [a,*], Simon A. Neild [a], Jonathan E. Cooper [b]

[a] Department of Mechanical Engineering, University of Bristol, Queens Building, University Walk, Bristol BS8 1TR, UK
[b] Department of Aerospace Engineering, University of Bristol, Queens Building, University Walk, Bristol BS8 1TR, UK

ARTICLE INFO

Article history:
Received 13 August 2014
Received in revised form
24 February 2015
Accepted 5 March 2015
Handling Editor: L.N. Virgin
Available online 26 March 2015

ABSTRACT

Backbone curves can offer valuable insight into the behaviour of nonlinear systems along with significant information about any coupling between the underlying linear modes in their response. This paper presents a technique for the extraction of backbone curves of lightly damped nonlinear systems that is well suited for the experimental investigation of structures exhibiting nonlinear behaviour. The approach is based on estimations of the instantaneous frequency and the envelope amplitude of a decaying response following a tuned steady-state oscillation of the system. Results obtained from simulations and experiments demonstrate that the proposed procedure is capable of achieving an accurate estimation of the backbone curves and damping ratios of the system provided that the premise of damping having low impact on its oscillation frequency is met.

1. Introduction

There has been much recent interest in predicting the behaviour of structures containing nonlinearities. This is, in part, the result of the latest developments in new materials together with the increasing computational capabilities that have brought to light a variety of novel design solutions to diverse engineering problems. For instance, a number of innovative structures, such as the civil aircraft A380 XWB and 787 Dreamliner, are notably more efficient and lightweight. Such systems have less inherent damping and are particularly flexible and therefore more susceptible to nonlinear effects. Consequently, the understanding of nonlinear dynamic systems and their performance in operational and under extreme loading conditions is an increasingly important research topic with potentially strong impact in many industrial sectors.

The presence of nonlinearities can be detected via standard techniques in experimental testing [1,2]. For instance, the variation of conventional Frequency Response Functions (FRF) obtained for different constant excitation levels, and also differences in the reciprocity between pairs of driving point FRFs, can be judged as clear evidence of nonlinear behaviour. However, the localisation, characterisation and quantification of such nonlinear behaviour are still open research topics [3]. A tool capable of offering a better understanding of the behaviour of nonlinear systems is the backbone curve [2,4] which defines the natural frequency as a function of the amplitude of the system response when neither damping nor forcing are present. The backbone curves provide a valuable description of the system dynamics that may allow for characterising and quantifying active nonlinearities whilst highlighting the interactions that may occur within the system, enabling for instance, the study of modal energy exchange due to nonlinearities, that cannot be analysed by conventional linearised

* Corresponding author.
E-mail addresses: Julian.Londono@bristol.ac.uk (J.M. Londoño), Simon.Neild@bristol.ac.uk (S.A. Neild), J.E.Cooper@bristol.ac.uk (J.E. Cooper).

http://dx.doi.org/10.1016/j.jsv.2015.03.015
0022-460X/

methods. In addition, it opens up the possibility of using backbone curves to identify or update nonlinear characteristics within a model based on the experimental response [5,6].

A technique for extracting backbone curves consists in the estimation of both the instantaneous amplitude and frequency along a free vibration response of the nonlinear system. In this context, a key approach is based on the application of the Hilbert transform to free vibration data, see for example the significant contribution made by Feldman [7]. Therein, a set of equations are proposed to estimate instantaneous characteristics such as amplitude, frequency and damping. Feldman extended and improved the method in [8], where the identification of nonlinear elastic forces acting in asymmetric systems was studied, leading to an effective identification. However, as discussed later, high-frequency superharmonics components of the Hilbert transform can be very sensitive to noise and so be detrimental to its estimation capabilities. Another approach is based on the use of Wavelets transform; in [9], a method for the identification of damping in MDOF systems was proposed by examining the system impulse response decomposed into the wavelets' time-scale domain. The method was further developed to extract ridges and skeletons of the Wavelets transform using optimisation algorithms based on simulated annealing [10]. These features were then employed to obtain the system backbone curve with a view to identify the linear and nonlinear parameters.

It is of interest to investigate structures that are at a first approximation linear at low vibration levels, but contain active nonlinear elements which become significant at larger excitation levels; typical of many industrial type structures. The interest here is to study the free vibration response originated from initial conditions that lie on a particular steady-state response of the nonlinear system. The Resonance Decay Method (RDM) [11] may be used, as this enables the excitation of modes of the system independently. Once the structure is vibrating at the desired resonance condition, the forcing is removed and the resulting free vibration response analysed. This strategy has proven to be able to isolate distinctive characteristics of several nonlinear systems through the case of fitting system parameters to the transient response [12–14].

In this paper, the RDM method is modified to estimate backbone curves for nonlinear systems. A procedure for the estimation of instantaneous amplitude, frequency and damping from decaying responses originated from steady-state oscillations is proposed. The approach enables measurements to be made in regimes of large displacements, where nonlinearities are more active. This facilitates more information to be deduced from the backbone curves, thus potentially enabling a realistic identification of the nature of nonlinearities. The approach is validated using a number of simulated systems with nonlinearities typically encountered in common engineering applications. Additional results are also included to show the applicability of the procedure on real experiment data.

This paper is organised as follows. Section 2 introduces the procedure proposed in this work. Simulated single-degree-of-freedom (SDOF) systems with nonlinear elements are used in Section 3 to show how backbone curves are estimated. The procedure is then validated using experimental data in Section 4. The following section presents the extension of this procedure to multi-degree-of-freedom (MDOF) systems, finishing with the final comments and remarks in the conclusions.

2. Estimation of backbone curves

In this section a method for measuring the backbone curves from appropriate structural responses and then for extracting frequency and damping information are discussed.

2.1. Obtaining the decay response

Transient responses contain information about all of the underlying fundamental features of dynamical systems, including those properties that are susceptible to change as a function of the oscillation amplitude. In particular, the interest is to examine free vibration records originated when setting the system free after obtaining a desired resonant response under harmonic forcing.

Consequently, in this procedure the signal used to extract the backbone curves of the nonlinear system is generated in accordance with the Resonance Decay Method (RDM) [11]. In this technique, individual modes of the system can be excited independently by applying an appropriated force pattern previously estimated. Such a force pattern is determined by using the normal-force mode appropriation method, that enables for extracting the undamped natural frequency and normal-modes shapes of a structural system [15]. After the appropriated force pattern is computed, this is applied to the system at the relevant frequency using harmonic excitation. When the structure is responding at the desired resonance condition, the input is removed and the model undergoes free vibration from the resonant response. As long as the level of vibration in the steady state is large enough to activate the structural nonlinearities, the generated decaying response offers significant information about the system and related parameters.

Once the decay response of the system is obtained, two main features, namely instantaneous frequency and amplitude envelope, can be estimated in the interest of extracting the backbone curves of the system.

2.2. Instantaneous frequency assessment

Whilst there are many procedures for calculating instantaneous frequency, such as the Wigner–Ville distribution (WVD) [16] and the Hilbert transform (HT) [7], the process presented here is based on the detection of the zero-crossing points of the response signal and the use of a standard interpolation algorithm to determine the crossing times. Noisy signals are