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A micro-inertia gradient visco-elastic motivation for proportional damping

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ABSTRACT

In this paper, a micro-inertia gradient visco-elasticity theory is proposed and implemented for the description of wave dispersion in periodic visco-elastic composites characterised by (stiffness-)proportional damping. An expression for the internal length parameter has been derived in terms of geometry and material properties. The theory has been validated through a numerical simulation of wave propagation in a one-dimensional periodic composite bar for two different heterogeneity levels, where the proposed theory has shown good agreement with the solution obtained by explicitly modelling the material heterogeneity. The effects of both gradient enrichment and viscosity on wave propagation as well as their interaction have also been analysed.

1. Introduction

Many gradient-enriched theories have been proposed in the past to overcome deficiencies of classical elasticity (see for instance [1–3]) and plasticity [4–7] theories in describing particular phenomena in both static and dynamics, such as size effects, strain and stress fields in the neighbourhood of crack tips and dislocation lines [2,3,8–12] and wave dispersion in dynamics [13–16].

The failure of classical theories in the description of the previously mentioned phenomena is due to the absence of internal length parameters, characteristic of the underlying microstructure. On the contrary, gradient-enriched theories are capable of considering the effects of microstructure on the macroscopic behaviour of a material by including high-order gradients, accompanied by internal length parameters, which are representative of the microstructure. Despite the significant number of gradient-enriched theories for elasticity and plasticity, gradient visco-elasticity theories have received less interest in the past. In particular, materials like synthetic polymers, biopolymers, wood, human tissues, bituminous materials and metals at high temperature show strong visco-elastic behaviours, thus it is of interest to develop a continuum theory that captures the micro-structural as well as the time-dependent phenomena.

An attempt to describe viscoelastic materials has been made by Gudmundson [17] in 2006, who proposed a strain gradient visco-elastic model to describe length scale effects in such materials. However, as the author points himself further in [17], the proposed method is lacking a link between the length scale and the material's microstructure.

In this paper a micro-inertia gradient visco-elasticity theory is proposed to study wave dispersion in periodic composites; allowing new insights about the effects of both gradient enrichment and viscosity on wave propagation as well as their

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interaction. The proposed theory is characterised by a direct link with the underlying microstructure as well as material properties. Moreover, an effective and straightforward finite element implementation of the presented theory is proposed. In Section 2, derivation of the continuum high-order model, characterised by an internal length parameter, is presented starting from a simple periodic discrete lattice model. In Section 3 the higher accuracy of the proposed methodology in capturing the dispersive behaviour of the discrete model is shown through a dispersion analysis. Thereafter, in Section 4 a brief description of the finite element implementation of the proposed theory is provided, along with some details about the adopted time integration algorithm. In Section 5 a homogenisation approach for periodic composites is proposed, along with a relation for the internal length scale, linking geometry and material properties. In Section 6 an application of the new methodology is presented, in order to show the interaction between viscosity and inertia gradients, as well as the different effects they have on wave propagation and dispersion. Finally, some final considerations about the gradient visco-elastic theory are given in Section 7, followed by a discussion of future research directions.

2. Model derivation

Gradient visco-elasticity theory can be obtained through the continualisation of a discrete lattice (Fig. 1) as explained in [18], taking into account that now the particles of mass m are in series with Kelvin–Voigt models characterised by spring stiffness s and viscosity η; the distance between two consecutive particles is denoted d. The equation of the motion of the n^{th} particle can be written as

$$m\ddot{u}_n = s(u_{n+1} - 2u_n + u_{n-1}) + \eta(\dot{u}_{n+1} - 2\dot{u}_n + \dot{u}_{n-1}) \tag{1}$$

Passing now from the discrete to the continuum model, the displacements can be rewritten with the following expressions:

$$u(x,t) = u_n(t) \quad \text{and} \quad u(x \pm d, t) = u_{n\pm 1}(t) \tag{2}$$

Then, by using the Taylor series the generic displacement can be written as

$$u(x \pm d, t) \approx u(x,t) \pm du'(x,t) + \frac{1}{2}d^2u''(x,t) \pm \frac{1}{6}d^3u'''(x,t) + \frac{1}{24}d^4u''''(x,t) \pm \cdots \tag{3}$$

and substituting Eq. (3) into Eq. (1), a new equation of the motion is obtained, in which a higher order term appears for both displacements and velocities:

$$m\ddot{u}(x,t) = sd^2\left[u''(x,t) + \frac{1}{12}d^2u''''(x,t)\right] + \eta d^2\left[\dot{u}''(x,t) + \frac{1}{12}d^2\dot{u}''''(x,t)\right] \tag{4}$$

or similarly, replacing the mass, the spring stiffness and the (stiffness-) proportional damping, respectively, with the following relations $m = \rho Ad$, $s = EA/d$ and $\eta = \tau EA/d$

$$\rho\ddot{u}(x,t) = E\left[u''(x,t) + \frac{1}{12}d^2u''''(x,t) + \tau\left(\dot{u}''(x,t) + \frac{1}{12}d^2\dot{u}''''(x,t)\right)\right] \tag{5}$$

where ρ is the mass density, E is the Young's modulus and τ is the (stiffness-) proportional damping coefficient. In Eqs. (4) and (5), truncation of the Taylor series has been applied such that only the next-highest order terms are maintained.

In this way an enriched-gradient model with positive sign has been obtained, which unfortunately has been proved to produce unstable results [19]. On the other hand, as shown in [19] the same model but with negative sign leads to stable results. This can be obtained through a simple mathematical manipulation that consists in taking the Laplacian of the original equation of the motion (5), that is (ignoring the spatial and temporal dependence for notational simplicity)

$$\rho\ddot{u}'' = E\left[u'''' + \frac{1}{12}d^2u'''''' + \tau\left(\dot{u}'''' + \frac{1}{12}d^2\dot{u}''''''\right)\right] \tag{6}$$

multiplying Eq. (6) by $\frac{1}{12}d^2$, which leads to (neglecting higher-order terms)

$$\frac{1}{12}d^2\rho\ddot{u}'' = \frac{1}{12}d^2E\left(u'''' + \tau\dot{u}''''\right) \tag{7}$$

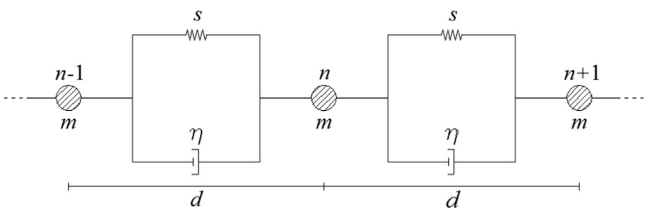

Fig. 1. Mono-dimensional discrete model consisting in particles and Kelvin–Voigt models.