Journal of Sound and Vibration 344 (2015) 81–100

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

The estimation of time-invariant parameters of noisy nonlinear oscillatory systems

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ARTICLE INFO

Article history:
Received 1 November 2013
Received in revised form
2 September 2014
Accepted 2 October 2014
Handling Editor: K. Worden
Available online 16 February 2015

ABSTRACT

The inverse problem of estimating time-invariant (static) parameters of a nonlinear system exhibiting noisy oscillation is considered in this paper. Firstly, a Markov Chain Monte Carlo (MCMC) simulation is used for the time-invariant parameter estimation which exploits a non-Gaussian filter, namely the Ensemble Kalman Filter (EnKF) for state estimation required to compute the likelihood function. Secondly, a recently proposed Particle Filter (PF) (that uses the EnKF for its proposal density for the state estimation) has been adapted for combined state and parameter estimation. Numerical illustrations highlight the strengths and limitations of the MCMC, EnKF and PF algorithms for time-invariant parameter estimation. For low measurement noise and dense measurement data, the performances of the MCMC, EnKF and PF algorithms are comparable. For high measurement noise and sparse observational data, the EnKF fails to provide accurate parameter estimates. Hence the adapted PF algorithm becomes necessary in order to obtain parameter estimates comparable in accuracy to the MCMC simulation with EnKF. It highlights the fact that the augmented state space model for the combined state and parameter estimation contains stronger nonlinearity than the original state space model. Hence the EnKF effectively handles the state estimation of the original state space model, but it fails for the combined state and parameter estimation using the augmented system. The effectiveness of the EnKF for the state estimation is therefore leveraged in the MCMC simulation for the time-invariant parameter estimation. In order to obtain accurate parameter estimates using the augmented system, the adapted PF becomes necessary to match the parameter estimates obtained using the MCMC simulation complemented by EnKF for likelihood function computation.

1. Introduction

In engineering systems, noisy oscillation arises in numerous aero-elastic and hydro-elastic problems (e.g. [1–3]). The noisy oscillation phenomenon received widespread attention in the structural dynamics research community as it may lead to large amplitude response and fatigue leading to system failure. The state and parameter estimates of systems exhibiting

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http://dx.doi.org/10.1016/j.jsv.2014.10.002
0022-460X/

noisy oscillation obtained from noisy observational data provide valuable information for assessing the safety and reliability of the engineering system in its operational state. In this paper, the problem of time-invariant (static) parameter estimation of a nonlinear system displaying noisy oscillation is considered blending informations contained in the observational data with the predictive model.

The parameter estimation problem of noisy oscillation falls in the general category of system identification [4]. In the context of nonlinear structural modelling, one of the earliest attempts of system identification was made by Ibanez [5] and Masri and Caughey [6]. Various techniques have been subsequently developed to tackle nonlinearity. For a thorough overview, the reader may refer to the book by Worden and Tomlinson [4] and review article by Kerschen et al. [7]. For a Duffing oscillator, Aguilar-Ibanez et al. [8] recently utilized an algebraic approach to the identification of the system parameters. Narayanan et al. [9] applied a hybrid time/frequency-domain-based Fourier series identification method to estimate the Duffing system parameters. The book by Bendat [10] provides a detailed exposition of Volterra series based nonlinear system identification techniques. The spectral identification procedures for nonlinear systems are reported by Zeldin and Spanos [11] and Spanos and Lu [12]. Kougioumtzoglou and Spanos [13] developed identification techniques for nonlinear systems based on harmonic wavelets. Manohar and Roy [14] estimated the state and nonlinear stiffness parameter of the Duffing oscillator from noisy observations using the PF with a Gaussian proposal distribution. In this paper, we continue our previous investigations [15,16] by applying EnKF, and PF for combined state and parameter estimation of a Duffing system.

Kalman Filter (KF) is generally applied to estimate the mean and covariance of the state based on observational data [17]. KF provides an optimal estimate for linear systems with additive Gaussian noise. For weakly nonlinear systems, KF may still provide reasonable estimates using linearization techniques leading to the so-called Extended Kalman Filter (EKF) [18–20]. The major limitation of EKF is due to the third- and higher-order moments in the error covariance evolution equation being discarded, leading to its poor performance [20]. While dealing with strongly nonlinear systems, Monte Carlo based sequential filtering algorithms have gained popularity due to their superiority over EKF. These sampling-based methods represent the probability density function (pdf) of the state vector using a finite number of randomly generated states. Typical examples of Monte Carlo based filters include EnKF [20] and PF [21–23]. EnKF effectively resolves some major problems encountered in EKF, including poor error covariance evolution [24]. Using a non-parametric approach, the nonlinear system identification technique is developed by using EnKF in structural dynamics [25]. EnKF has been shown to perform poorly in some applications involving highly non-Gaussian system behaviour [26,27]. As a remedy to this problem, a filter has been proposed by Anderson [28] that utilizes a weighted sum of Gaussian pdfs to represent the pdf of the system state. For time-invariant parameter estimation using PF for real time applications, Storvik [29] developed a methodology to marginalize the static parameters from the posterior when the pdf of the unknown parameters lies on some low-dimensional sufficient statistics. Storvik's approach [29] avoids the sample impoverishment in PF while estimating time-invariant parameters. For static parameter estimation, Vrugt et al. [30] used the Particle Markov Chain Monte Carlo method [31] which uses PF to design efficient proposal density for MCMC.

PF [21–23] is a Bayesian data assimilation technique that makes neither Gaussian assumption nor linearization of the model and measurement operators. For highly nonlinear systems, PF may provide better estimates compared to EnKF (e.g. [15,16]). For joint state and parameter estimation, PF usually requires an extremely large ensemble of particles (or Monte Carlo samples) compared to EnKF in order to avoid filter divergence in PF [20,32]. The required ensemble size may be reduced in PF through a resampling step [14,23,33,34] in conjunction with an efficient sampling technique, such as Latin Hypercube Sampling (LHS) [35,36], as shown in previous investigations by the authors [15,16]. To further alleviate the requirement of large ensembles, an artificial dynamic model of the unknown parameters is introduced by which the parameters are modelled as Wiener processes [37,38], but the performance of this approach is rather poor [39]. More recently, several investigators [40,33,39] have proposed regularization of the distribution of the state vector. However, the regularization step utilizes convolution kernels which have an effect similar to that caused by the introduction of artificial dynamics [39,38]. In addition to these proposed strategies, a number of PF algorithms have been proposed that utilize other nonlinear filters in providing more efficient proposal distributions, leading to the so-called EKF–PF and Unscented Kalman Filter (UKF)–PF (e.g. [41]) and the EnKF–PF [42,43].

In [42,43], the (non-Gaussian) proposal is obtained with kernel density estimation (KDE) [44] applied to the updated EnKF ensemble using the distance in Sobolev spaces. More specifically, the norm of the Cameron–Martin space [45,46] is used for deriving the density estimates. For the joint state and parameter estimation using nonlinear filters, the state vector is augmented by the unknown parameters to be estimated. The norm associated with the Cameron–Martin space is no longer applicable to this augmented vector since it can no longer be considered as a discrete representation of smooth functions. This PF algorithm will be extended in this paper to deal with the estimation of time-invariant parameters. The authors propose the application of a generalization of Scott's rule for multivariate KDE as in [44,47]. This step consists of applying a Mahalanobis transformation [48] to the augmented state vector to transform the estimated covariance matrix of the augmented state vector to identity. KDE will subsequently be performed using this transformed vector using a generalization of Scott's rule [44,47] for the multivariate case. Finally, the estimated pdf will be transformed back to the original coordinate. The proposed PF circumvents the need to artificially inflate the variances of the parameters in order to avoid filter divergence, thus making it well-suited for the time-invariant (i.e. static or fixed) parameter estimation.

For the parameter estimation of dynamical systems using nonlinear filters, the unknown system parameters are concatenated to the state vector leading to a higher dimensional state space model than the original system. Even if the