Journal of Sound and Vibration 344 (2015) 209–220

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Dynamics of Timoshenko beam on linear and nonlinear foundation: Phase relations, significance of the second spectrum, stability ☆

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ARTICLE INFO

Article history:
Received 22 September 2014
Received in revised form
7 January 2015
Accepted 21 January 2015
Handling Editor: W. Lacarbonara
Available online 10 February 2015

ABSTRACT

Some controversial and unclear issues in the Timoshenko beam (TB) theory are analyzed with emphasis on features of two branches of frequencies. Taking into account the elastic foundation creates additional opportunities for comparative analysis of both the branches. Relationship between the second branch modes and so-called “thickness-shear mode” is discussed. Phase relations between the deformation and force factors in beam for both the branches are established which generalize the previously established relations. It is shown that the elastic foundation can change these relations for the first branch. Based on this analysis the significance of the second spectrum is assessed, and peculiarities of dispersion relations for frequency, phase and group velocities are explained.

Free transverse waves in infinitely long beam on nonlinear (cubic) foundation are studied using the multiple scales method. The obtained nonlinear Schrödinger equation enables us to check stability of the harmonic waves in infinite beam and to determine diapasons of their modulational instability.

1. Introduction

The Timoshenko beam model which takes into consideration the shear flexibility of beam and the rotatory inertia [1] has become popular for two main reasons. First, it can yield considerable refinement to the classical Euler–Bernoulli (E–B) theory (as well as to the Rayleigh (R) theory accounting the rotatory inertia but not the shear). Second, the TB model results in a wave type equation in dynamics, in contrast to the E–B model yielding to a non-wave type dynamic equation [2,3].

The principal problems of the Timoshenko beam theory (TBT) have been studied in 1950–1960s (see, e.g., review [4] and papers [5–7]). From the mathematical viewpoint, the main feature of the TBT is that the stress–strain state at a point is defined by a two-dimensional vector (e.g., total deflection and bending slope), in distinction on a scalar (deflection) in the E–B and R models. A principal consequence is the existence of two frequency spectra (or two branches in case of an infinite beam), with different phase relations between two components of this vector (different phases of the bending and shear angles) [5]. It has been understood that accounting both the branches is necessary for solution of an initial-value problem since it requires superposition of the both branch modes [6,7]. The correct orthogonality conditions for natural modes have

☆ This work has been partially reported on the 8th European Nonlinear Dynamics Conference (July 6–11, 2014, Vienna)

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http://dx.doi.org/10.1016/j.jsv.2015.01.027

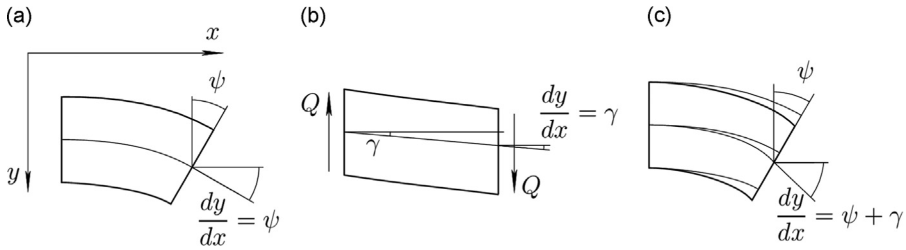

Fig. 1. Bending (a), shear (b) and total (c) deformation.

been established taking into account the two-dimensional character of TBT solutions [6,7]. There was also detected a special solution – so called "thickness-shear mode" with a "cut-off frequency" which does not depend on wavelength and separates two spectra [8].

But later, in 1970–1980s, the meaning and significance of the TB second branch were questioned in a series of papers ([9–12] and others). Different reasons were put forward for recognition of the second spectrum "unphysical". First, the second spectrum can be easily isolated and detected only in cases of those boundary conditions for which the frequency equation factorizes. Second, numerical comparison of the second spectrum predictions (TBT2) to the exact plane stress elastodynamic theory has shown that TBT2 predictions are not in consistent agreement with any single mode of vibration obtained in the exact theory (good agreement was noted with different modes depending on the wavelength). These and others arguments were challenged by other authors [13]. Some researchers asserted that there is no need to refer to TB frequencies as separate frequency spectra ("the single-frequency interpretation", [14] and others).

Debate on this topic continued in the following decades (here we may refer to a detailed historical review in paper by Stephen [15]). The views expressed in papers [15,16], which can be considered as the last word on this subject, may be reduced to the following statements:

(1) The single-spectrum interpretation is not justified in view of qualitative differences between the two spectra modes ("If, above some cut-off frequency, there are new modes of vibration whose character differs in some way from what one is familiar with below this cut-off (the first spectrum) then it is entirely legitimate to describe the new modes as a *second* spectrum, irrespective of whether the frequency equation factorizes" [15]).
(2) "Timoshenko predictions above the cut-off frequency should be disregarded for those end conditions for which the frequency equation does not factorize" [16].

As the TB second spectrum always lies above the cut-off frequency, the later conclusion, in fact, means that the second spectrum is meaningless. Note that statement (2) is partially contrary to the statement (1) as it attaches fundamental importance to the factorization of the frequency equation.

From the general theoretical viewpoint, with account of two-dimensional character of TBT solutions, statement (2) is at least doubtful. The reasons for neglecting the second spectrum are based on results of numerical experiments [10,15,16] which may be interpreted differently.

In this paper we consider free waves and oscillations in Timoshenko beam on elastic (Winkler) and nonlinear (cubic) foundation focusing on features of the both frequency branches. For simplicity we restrict ourselves with infinite beams, this facilitates proofs of the main assertions. The presence of foundation creates additional challenges and opportunities for comparative analysis of the both frequency branches. Relationship between the second branch modes and so-called "thickness-shear mode" is discussed. We show that qualitative differences between the two branches are specified by the phase relations between the total, bending and shear slopes (as well as between the force factors) which become more complicated in the presence of a foundation and for the first time are presented in the paper. Based on this analysis the significance of the second spectrum is assessed, and peculiarities of dispersion relations for the frequency, phase and group velocities are explained. In the last section free transverse waves in infinitely long beam on the nonlinear (cubic) foundation are studied using the multiple scales method, and stability of the harmonic waves is testified by means of the nonlinear Schrödinger equation.

2. Governing equations and some relations for harmonic waves and oscillations

In the TBT the total slope of the bent axis ($\partial y/\partial x(y(x,t)$ is the transverse displacement, x is the longitudinal coordinate) is equal to sum of the cross section rotation angle ψ (due to the bending) and the shear angle γ: $\partial y/\partial x = \psi + \gamma$ (Fig. 1). So, in distinction to the classical models (E–B, R) where the stress–strain state is entirely defined by the function $y(x,t)$, the deformation in TB is defined by a two dimensional vector (y, ψ) (or by other two independent variables, e.g., bending and shear deflections).[1]

[1] It devalues attempts to construct the TBT on the basis of a functional depending on a single function, see, e.g., [12].