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Design method of planar vibration system for specified ratio of energy peaks

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ABSTRACT

The magnitudes of the resonant peaks should be considered in the design stage of any bandwidth-relevant applications to widen the working bandwidth. This paper presents a new design method for a planar vibration system that satisfies any desired ratio of peak magnitudes at target resonant frequencies. An important geometric property of a modal triangle formed from three vibration centers representing vibration modes is found. Utilizing the property, the analytical expressions for the vibration energy generated by external forces are derived in terms of the geometrical data of vibration centers. When any desired ratio of peak magnitudes is specified, the locations of the vibration centers are found from their analytical relations. The corresponding stiffness matrix can be determined and realized accordingly. The systematic design methods for direct- and base-excitation systems are developed, and one numerical example is presented to illustrate the proposed design method.

1. Introduction

The magnitude of the resonant peak is an important factor which should be considered in the design stage of a vibration system. The importance of this factor becomes especially apparent for applications that are relevant to the bandwidth, such as vibration-based energy harvesters or vibration absorbers. For example, although the natural frequencies of multimodal energy harvesters are designed to be close to one another, a wide bandwidth cannot be guaranteed if the peak magnitudes are not uniform.

While the peak magnitude of a 1-degree of freedom (dof) system can be adjusted easily by tuning the parameters, it is not easy to tune the peak magnitudes of multimodal systems. This is because the complexity increases as the magnitude of the resonant peak of a multimodal system depends on both the resonant frequency and the vibration mode. Furthermore, the vibration modes are constrained by the orthogonality property with respect to the inertia matrix in general, which increases the complexity.

To tune the peak magnitudes of multimodal systems, optimization methods are widely used. However, these numerical methods lack the clear physical meaning of the system parameters, and often fail to give proper solutions. Many studies on the peak magnitudes for multimodal systems have been made to overcome such problems, especially in the field of energy harvesters. One approach is to use a multiple mass array [1–4]. Because the total system consists of independent 1-dof

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systems, the magnitudes of the peaks can be adjusted independently with that method. A bulky system is needed to obtain a wide bandwidth from independent multiple peaks without using the coupling effect. Tang and Yang analyzed the peak magnitudes of a multimodal energy harvesting system that consisted of serially stacked 1-dof systems [5]. In that research, the coupling effect between the primary and stacked bodies was explained analytically. Since a rigid body has 6-dofs (3-dofs for the planar case) in space, it is desirable to use as many dofs as possible for greater efficiency. For that purpose, Jang et al. derived an analytical expression of the magnitudes of resonant peaks by utilizing the impedance method for a simple 2-dof model which consisted of a single mass and two parallel supporting beams [6]. Doğan proposed a serial type energy harvester which is a two-link flexible arm with non-uniform cross-section [7]. He showed that the combination of the concepts of nonlinearity from variable beam geometry [8] and multi-degrees of freedom is efficient in broadening bandwidth.

Though there has been much research in this area, a design method for a more general system is still required. The simplest case would be one in which all the modes are decoupled. In such a case, the magnitudes of modes may be adjusted independently. However, if a pure force is applied to the body of such a decoupled system, pure rotational modes cannot be excited. On the contrary, a pure moment cannot excite pure translational modes. Therefore, to make a system have the desired resonant frequencies and magnitudes of resonant peaks for general excitation, the coupling relationship between vibration modes should be considered together. Blanchet first introduced the geometrical relationship between vibration modes via screw theory [9]. He showed that the triangle formed by three planar vibration modes has its orthocenter at the mass center of the system. Dan and Choi derived the analytical expression of vibration mode for the system that has planes of symmetry [10]. They also designed an optical pickup device using the root locus representing the variation modes for a spatial system with one plane-of-symmetry [11]. Recently, the geometric properties of the modal triangle such as the area and shape were further investigated by Jang et al. [12].

This paper presents a new design method for a planar vibration system with given mass properties (mass and moment of inertia) that can be used to determine the desired ratio of vibration energies at specified resonant frequencies. It is described in the next section that three vibration modes represent the centers of vibration, which form a triangle with orthocenter at the mass center. The triangle is referred to hereinafter as a modal triangle. The proposition of a modal triangle, which states that it becomes an acute triangle, is given. In Section 3, the proposition is used to derive the analytical expressions for vibration energy induced by an external force in terms of the geometrical data of the vibration modes. The systematic design methods for both direct- and base-excitation systems are described. The final section illustrates one numerical design example.

2. Theoretical preliminaries on modal triangle

When a rigid body is elastically suspended in a plane, the equation of motion for undamped free vibration at any coordinate frame A is given by

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{K}\mathbf{X}=\mathbf{0}, \tag{1}$$

where $\mathbf{M},\ \mathbf{K} \in R^{3\times 3}$ are the inertia and stiffness matrices, respectively. The general form of the displacement vector $\mathbf{X}$ can be expressed by

$$\mathbf{X}=\hat{\mathbf{X}}e^{j\omega t}, \tag{2}$$

where ω denotes the natural frequency of the system and $\hat{\mathbf{X}}$ is time-independent.

In Eq. (2), $\hat{\mathbf{X}}$ can be given by

$$\hat{\mathbf{X}}=\begin{bmatrix}\delta x_o & \delta y_o & \delta\varphi\end{bmatrix}^T, \tag{3}$$

where δx_o and δy_o represent the x- and y-components of small translational displacement of a point on the rigid body coincident with the origin, $\delta\varphi$ is the angle of small rotational displacement as shown in Fig. 1. By dividing $\hat{\mathbf{X}}$ by $\delta\varphi$, the normalized vector representing the line parallel to the z-axis and passing through the instantaneous center of motion can be expressed by [13]

$$\hat{\mathbf{S}}=\begin{bmatrix}y & -x & 1\end{bmatrix}^T, \tag{4}$$

where x and y are the coordinates of instantaneous center of motion. The planar vibration motion can also be expressed by such representation, and three normal modes of vibration that are solutions of Eq. (1) can be expressed by

$$\hat{\mathbf{S}}_i=\begin{bmatrix}y_i & -x_i & 1\end{bmatrix}^T, \quad (i=1,2,3), \tag{5}$$

where $(x_i,\ y_i)$ are the coordinates of the vibration center of the ith normal mode. Normal modes of vibration are orthogonal to each other with respect to the inertia and stiffness matrices, and it can be written as

$$\mathbf{S}^T\mathbf{M}\mathbf{S}=\begin{bmatrix}\tilde{m}_1 & 0 & 0\\ 0 & \tilde{m}_2 & 0\\ 0 & 0 & \tilde{m}_3\end{bmatrix} \text{ and } \mathbf{S}^T\mathbf{K}\mathbf{S}=\begin{bmatrix}\tilde{k}_1 & 0 & 0\\ 0 & \tilde{k}_2 & 0\\ 0 & 0 & \tilde{k}_3\end{bmatrix}, \tag{6}$$