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The full nonlinear crack detection problem in uniform vibrating rods

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A R T I C L E I N F O

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A B S T R A C T

The basic problem in structural diagnostics via dynamic methods consists in determining the position and severity of a single open crack in a beam from the knowledge of a pair of resonant or antiresonant frequencies. A well-established theory is available for longitudinally vibrating uniform beams when the severity of the crack is small. In this paper we fill the gap present in the literature by showing that the results of the linearized theory for slight damage can be extended to a crack with any level of severity.

1. Introduction

Dynamic testing is commonly used as a diagnostic tool to detect damage that has occurred in a mechanical system during service. The final goal is to predict location and level of severeness of the degradation from the measurement of the changes induced by the damage on the vibrational behavior of the system.

Within the large class of diagnostic problems arising in structural mechanics, the crack detection problem in vibrating beams by frequency data has received a lot of attention in the scientific community in last two–three decades, see, among other contributions, the research developed in [1] and [2] for an introduction to the topic and for recent advances on multiple crack identification in beams, respectively. The reasons for this interest are various. Firstly, the beam model describes the behavior of structural members that play an important role in many civil and mechanical engineering applications. Secondly, the problem of identifying a crack in a beam is the basic diagnostic problem and, therefore, it represents an important benchmark to test the effectiveness of damage identification techniques. In addition, concerning the type of input data, in most applications researchers have used natural frequencies or antiresonant frequencies as an effective damage indicator. Frequencies can be measured more easily that mode shapes, and are usually less affected by experimental and modelling errors.

Among the various models that have been proposed in the literature to describe (open) cracks in beams, localized flexibility models enable one for simple and effective representation of the behavior of damaged elements [3]. The results of extensive series of vibration tests carried out on steel beams with a single and multiple cracks confirm that lower natural frequencies are predicted by localized flexibility models with accuracy comparable to that of the classical Euler–Bernoulli model for a beam without defects, see, for example, [4].

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In this paper we shall be mainly concerned with the inverse problem of identifying a single open crack in a longitudinally vibrating beam by frequency data. The crack is modelled by inserting a translational linearly elastic spring at the damage cross-section. On assuming that the undamaged configuration is completely known, the inverse problem consists in determining the location s of the crack, and its magnitude or severity. This latter parameter is expressed in terms of the stiffness K of the spring simulating the crack, and the undamaged configuration is obtained by taking the limit as the stiffness K tends to infinity or, equivalently, as the flexibility $1/K$ tends to zero.

When the crack is small, namely when the cracked rod is a perturbation of the undamaged rod, a well-established theory for the inverse diagnostic problem is available, see Gladwell [5, Chapter 15]. The cornerstone property concerns with the possibility to express the change $\delta(\lambda_n^2)$ of the (square of) nth natural frequency λ_n produced by a small single crack as a product of the flexibility $1/K$ and the square of the axial force $N_n^{(U)}(s)$ evaluated on the undamaged configuration, for the relevant mode shape, at the cracked cross section [6–8]:

$$\delta\left(\lambda_n^2\right) = -\frac{(N_n^{(U)}(s))^2}{K}. \tag{1}$$

Then, in the case of small crack, the ratios of the change in different natural frequencies depend on the damage location s only, not on the severity K. Hearn and Testa [1], Liang et al. [9], Rubio [10], among others, have used this property for damage localization in beam-like structures. See also Lakshmanan et al. [11] for identification of localized damage in rods from the iso-eigenvalue change contours constructed between pairs of different frequencies.

Concerning the rigorous, i.e., mathematically proved, identification of a small crack in an axially vibrating rod, worth of mention is the result obtained by Narkis [12]. Narkis proved that a single crack in a uniform free–free rod can be uniquely localized (up to a symmetric position) by using the first two natural frequencies. Working on a linearized version of the frequency equation, Narkis obtained a closed-form solution for the crack location s. Using relation (1), Morassi [13] extended Narkis's result to rods with single small crack under different sets of end conditions and for different pairs of natural frequencies, providing closed-form expressions also for the damage severity K. Later on, Dilena and Morassi [14] proved that the measurement of the first natural frequency and the first antiresonance of the driving point frequency response function evaluated at one end of a free–free uniform rod allows us to uniquely determine the position of the crack and its severity, by means of closed-form expressions. Extensions to cracked rods with dissipation [15], cracked pipes [16] and multi-cracked rods and beams [17,18] are also available.

All the above-mentioned results hold in the case of small damage. Therefore, an important question is left open: Can the above results be extended to the case of not necessarily small damage? More specifically, the following question can be posed:

(Q) *Do Narkis's result [12] and Dilena–Morassi's result [14] continue to hold even for a large crack*?

The a priori hypothesis of small crack is often considered to be a no very restrictive limitation, since in several practical situations it is of interest to be able to identify the damage as soon as it arises in a structure. However, there are several motivation in support of the opportunity to extend the theory to *not necessarily small* cracks. Firstly, it is not easy to determine rigorously when a crack can be considered small. The smallness of a crack is typically established on the basis of the crack-induced changes on the lower natural frequencies. However, this criterion is difficult to apply, since the vibration modes have different sensitivities to damage according to the position of the crack along the beam axis. The introduction of an "average" frequency shift does not simplify the analysis, since it should be clarified how many data must be included in the calculation and how the threshold value corresponding to small damage should be selected. Secondly, the linearized theories by Narkis and Morassi show some limitation when the damage is located near a point of vanishing sensitivity for a vibration mode. In [13] it was shown that this aspect also prejudices the reliability in assessment of the damage severity (see Table 3 in [13]). Analogous indeterminacy was recently encountered in identifying multiple small cracks in a longitudinally vibrating beam by frequency measurements [18]. Near a zero-sensitivity point, the first-order effect of the damage on the frequencies vanishes. Therefore, it is expected that the indeterminacy can be removed by considering the full nonlinear inverse problem instead of its linearized version. Finally, it is of course desirable to have a unifying general theory of the diagnostic problem capable to include damages ranging from small to large severity.

The waiver to the linearized theory implies strong consequences and, in particular, the inability to obtain an explicit relationship like (1) expressing the change of a natural frequency in terms of the damage parameters and the undamaged configuration. In the full nonlinear crack detection problem, the relationship between damage parameters (s,K) and natural frequencies is implicitly contained in the frequency equation of the cracked rod. Diagnostic methods based on the analysis of the frequency equation have been already investigated in the literature. The approach generally consists in considering a pair of frequency data (typically, the first two natural frequencies) and then solving *numerically* the nonlinear system formed by the frequency equation written for the two selected frequencies in terms of the damage parameters s and K. We refer, among others, to the pioneering research developed by Adams et al. [6], the studies by Springer et al. [19] and Lin and Chang [20] on longitudinally vibrating rods, and the contributions by Nandwana and Maiti [21], Cerri and Vestroni [22], Vestroni and Capecchi [23] on cracked beams in bending vibration. The above results seem to suggest that the answer to question (Q) is positive. However, at the best of our knowledge, a rigorous proof of this general property is not available, as the conclusions of the cited papers are drawn either on the basis of numerical analysis of specific cases or on the study of particular experimental situations.

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