



Free vibrations of a taut cable with a general viscoelastic damper modeled by fractional derivatives



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ABSTRACT

This study extends dynamic understanding of a taut cable with a viscous damper at arbitrary location to that with a general linear viscoelastic (VE) damper portrayed by a five-parameter fractional derivative model (FDM). The FDM is able to describe a generalized relationship between force and deformation of viscoelastic dampers (material) in a wide frequency range, which can simulate a practical damper including its support condition or a secondary tie between neighboring cables. Free vibrations of the passively controlled cable system have then been formulated analytically through complex modal analysis. For the restricted case that the FDM is installed close to one cable anchorage, asymptotic solutions for the system complex frequency and modal damping are presented; explicit formulas have also been derived to determine the maximal attainable damping and corresponding optimum FDM parameters, based on which effects of frequency-dependent damper properties are appreciated. Considering the FDM located at arbitrary location, the three distinct regimes of frequency evolutions observed for a cable with a viscous damper have been generalized to that with a VE damper; also, new characteristics of the regime diagram and the frequency evolution in each regime are observed.

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1. Introduction

Attaching mechanical dampers, especially of linear viscoelastic (VE) type, on stay cables in cable-stayed bridges has become a regular design for cable vibration mitigation due to its proven effectiveness, robustness and simpleness [1–3]. Meanwhile, understanding of the passively controlled cable system has gradually advanced in the past decades via adopting more and more detailed models in dynamic analyses: cable model has evolved from a taut string to a tensioned beam with static deformation; varied mechanical models consisting of elastic springs and dashpots have also been used to describe properties of the damper and its support condition, as illustrated in Fig. 1, where $c, c_0, \dots, c_q$ are coefficients of the dashpots; k is the constant of the spring in parallel with the dashpots; $b, b_1, \dots, b_q$ are relaxation time constants of springs in series with the dashpots; α and β are fractional derivative orders in the fractional derivative model (FDM).

According to the parameters included, previous studies on the cable/linear damper system can be summarized into categories listed in Table 1. For practical damper design, initial researches were dedicated to the dynamics of a taut string with a viscous damper near its anchorage, e.g. [4–10]. Further, more parameters pertaining to the cable, such as the

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Nomenclature		$u(t), \hat{u}(\omega)$	damper deformation and its Fourier transformation
A, B, C, D	quantities in expressions of the normalized storage and loss stiffness of a FDM	x, x_j	coordinate along chord of the cable and each segment
$b, b_1, \dots, b_q$	relaxation time constant in damper models	$y_j(x_j)$	lateral deflection of segment j
$c, c_0, c_1, \dots, c_q$	damping coefficient of dashpots in damper models	$Y_j(x_j)$	complex mode shape of segment j
$c(\omega)$	apparent damping coefficient of a FDM	α, β	fractional derivative order in a FDM
$\tilde{c}_0^{\text{opt}}$	optimum damper coefficient	$\Gamma(\cdot)$	gamma function
$\mathcal{D}[\cdot]$	fractional derivative operator	ζ_i	damping ratio of cable mode i
$f(t), \hat{f}(\omega)$	time function and its Fourier transformation	$\zeta_{i,\text{opt}}$	optimum damping ratio
$\mathcal{F}[\cdot]$	Fourier transform operator	θ_i	phase angle of eigenfrequency $\tilde{\lambda}_i$
$G_1(\omega)$	storage stiffness of a FDM	λ	dimensional cable eigenfrequency
$G_2(\omega)$	loss stiffness of a FDM	$\tilde{\lambda}_i$	dimensionless complex eigenfrequency
G_{1i}	normalized storage stiffness of a FDM	μ_j	dimensionless length of cable segment j
G_{2i}	normalized loss stiffness of a FDM	σ_i	real part of eigenvalue $\tilde{\lambda}_i$
i	cable mode number	τ	variable in the definition of fractional derivative
i	the imaginary unit $\sqrt{-1}$	φ_i	damped frequency, also the imaginary part of $\tilde{\lambda}_i$
j	cable segment index	$\varphi_{ci}^{(j)}$	i th clamped frequency of segment j
k	spring constant in damper models	$\varphi_{oi}^{(j)}$	undamped frequency of mode i
$\tilde{k}, \tilde{c}_0, \tilde{b}$	dimensionless k, c_0 and b normalized to cable fundamental frequency	$\varphi_{\text{crit},i}^{(j)}$	damped frequency of segment j in mode i corresponding to critical damping
l, l_j	cable/segment length	ω_i	modulus of dimensionless eigenfrequency $\tilde{\lambda}_i$
m	cable mass per unit length	ω_{01}	cable fundamental natural frequency
n	node number of cable mode shape	Ω_i	asymptotic cable eigenfrequency of mode i
$P(t), \hat{P}(\omega)$	damping force and its Fourier transformation	$\Omega_{i,0}$	cable eigenfrequency of mode i without damper
R_j, Q_j	quantities in the characteristic equation		
t	time		
T	cable tension		

inclination, sag [11–17] and bending stiffness [18–21], were taken into account. Recently, more sophisticated models to emulate the real damping material as well as the damper support have been adopted: damper intrinsic stiffness was considered by a Kelvin–Voigt model [22–25]; the support flexibility was addressed using a Maxwell model [25–28]; and they were treated together through a three-element standard mechanical model (SMM) [29] (generalized Maxwell model). A two-parameter fractional viscous damper was briefly considered in [22] to generalize the asymptotic format for the modal damping appreciation of a transverse damper close to the end of a cable.

The aforementioned literature focuses mainly on the restricted case that the damper location is in the vicinity of cable anchorage for traditional engineering demand. One issue that has not been fully considered thereof is the frequency dependence of damper properties, namely the rate-dependent relaxation time constant and apparent damping coefficient, which are of considerable significance because multiple cable modes are usually targeted [16,32] in practical damper design. To describe the broadband rheological behavior of a VE damper, even the three-parameter Maxwell model is inadequate, and more complicated models are required such as SMM and FDM with more elements. Actually, since the introduction of the notion of FDM for a generalized relationship between stress and strain of viscoelastic material [33,34], this model has been predominantly used. For instance, a four-parameter FDM was used to fit the viscoelastic properties of a damper consisting of a piston moving in a highly viscous gel in [35–37]. Predictions by the obtained model were in excellent agreement with experimental results over a wide frequency range. Applications of FDM to solid VE dampers can also be found [38]. A review on the performance of different mathematical modeling of VE dampers [39] has demonstrated that fewer variables are required for a FDM than a SMM to fit the experimental data while the modeling errors are of comparable level. Besides, characteristics of a cable/linear damper system are mostly discussed in frequency domain, the complicated

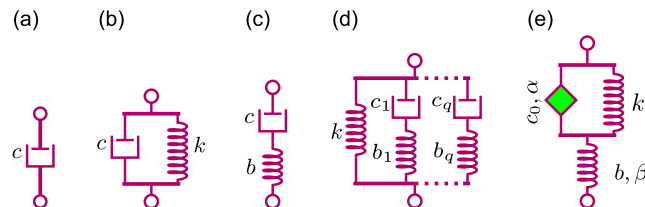


Fig. 1. Viscoelastic damper (material) models: (a) viscous; (b) Kelvin–Voigt; (c) Maxwell; (d) SMM; (e) FDM.

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