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Vibration suppression for strings with distributed loading using spatial cross-section modulation

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ABSTRACT

A problem of vibration suppression in any preassigned region of a bounded structure subjected to action of an external time-periodic load which is distributed over its domain is considered. A passive control is applied, in which continuous spatially periodic modulations of structural parameters are used as a means for vibration suppression. As an example, stationary vibrations of a string under action of a distributed time-periodic load are studied. This system in a simplified form models such processes as interaction between membranes and colloids, oscillations of transmission lines under action of rain and wind, and dynamics of suspension bridges and stay cables. Suppression of vibration in predefined regions of the string is performed by continuous spatial modulation of its cross-section.

For analyzing the problem considered a novel approach named the method of varying amplitudes is employed. This approach is applicable for solving differential equations without a small parameter, and may be considered as a natural continuation of the classical methods of harmonic balance and averaging. As a result, optimal parameters for the string cross-sectional area modulation are determined for the cases of harmonically, uniformly and arbitrarily distributed load, which allows for completely suppressing or considerably reducing vibration in the prescribed part of the string (compared to the case without modulation).

1. Introduction

The paper is concerned with a problem of vibration suppression in any preassigned region of a bounded structure subjected to action of an external time-periodic load which is distributed over its domain. Similar problems of sound and vibration isolation gained much attention in the recent years (e.g. [1–8]). The approaches proposed for treating such problems may be divided in two groups: active control, when structural parameters are varied in time [3,8–12], and passive control, when these parameters are changing only in space [2,5,7,13–15]. In most cases behavior is considered in presence of a point load or an excitation source, and boundary conditions are not taken into account [2–6,12–14,16,17]. Then the problem reduces to identifying frequency stop and pass bands, and their subsequent tailoring by spatial/temporal

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modulations of system parameters. In the present paper the problem of vibration suppression is considered in a different formulation: behavior of a bounded structure under action of a distributed load is studied. A passive control is applied, in that continuous spatially periodic modulations of structural parameters are proposed to be used as a means for vibration suppression. We note that the problem under study is closely related to those solved by the method of topology optimization [5], which is a popular method for obtaining an optimal layout of one or several material constituents in structures and materials. However, the important advantage of the analytical approach employed in the present paper over this and other numerical methods is the ability to provide insight into, e.g., explicit dependencies of structural dynamic characteristics on parameters.

As an illustrative example, consider vibrations of a string under action of an external distributed time-periodic load. Suppression of vibration in predefined regions of the string is performed by continuous spatial modulation of its cross-section. This simple system is a generic model for various relevant problems, e.g. interaction between membranes and colloids in biochemistry [18], oscillations of transmission lines under action of rain and wind [19], and dynamics of suspension bridges and stay cables [20]. Moreover, it serves to reveal general effects of continuous spatially periodic modulations on oscillations of bounded structures subjected to distributed loading, and illustrates possible advantages and disadvantages of the proposed technique of vibration suppression.

For analyzing the problem considered a novel approach named the *method of varying amplitudes* (MVA) is employed. This approach is inspired by the method of direct separation of motions (MDSM) [21,22], and may be considered a natural continuation of the classical methods of harmonic balance [23] and averaging [24–26]. It implies representing a solution in the form of harmonic series with varying amplitudes, but in contrast to the averaging methods the amplitudes are not required to vary slowly. Thus the MVA does not assume the presence of a small parameter in the governing equations, or any restrictions on the sought solution. This, in particular, makes it convenient for analysis of the considered problem which implies solving differential equations without a small parameter.

The paper is structured as follows: In Section 2 the equations of motion for the inhomogeneous string are presented, and the specific aims of the analysis are outlined. Section 3 is concerned with the case of *spatially harmonic external load*: In Section 3.1 a solution of the governing equations by the MVA is provided, and essence of the method is briefly described. In Section 3.2 optimal parameters of the string cross-sectional area modulation are determined that ensures complete suppression or considerable reduction of vibration in predefined regions of the string. Section 4 is concerned with the case of *spatially uniform external load*. In Section 4.1 the corresponding equations are solved by the MVA, while Section 4.2 calculates optimal parameters of the string cross-sectional area modulation. Finally, Section 5 is concerned with the case of *arbitrarily distributed external load*.

2. Governing equations

Consider vibrations of a string with variable cross-section under action of a distributed load which are described by the equation

$$\rho S(x)\frac{\partial^2 u}{\partial t^2} - T\frac{\partial^2 u}{\partial x^2} = f(x,t), \tag{1}$$

where ρ is density of the string material, T the tension force (assumed constant), $u(x,t)$ the lateral deflection of the string, $x \in [0;l]$ the axial coordinate, $S(x)$ the variable cross-sectional area, and $f(x,t)$ the external load per unit length. The boundary conditions are: $u(0,t) = u(l,t) = 0$, and l is length of the string; motion of the string is studied at zero initial conditions. Dissipation is assumed to be negligibly small. The case of a time-varying load of period Θ is considered:

$$f(x,t) = f(x,t+\Theta).$$

This is justified and common for most of the abovementioned problems; e.g. impact of rain and wind on transmission lines and stay cables [19,20], and colloids on membranes [18] is usually modeled as time-periodic. Expanding f in a trigonometric Fourier series gives:

$$f(x,t) = c_0(x) + \sum_{n=1}^{\infty}\left(c_n(x)\cos\left(\frac{2\pi}{\Theta}nt\right) + d_n(x)\sin\left(\frac{2\pi}{\Theta}nt\right)\right). \tag{2}$$

Consequently, since (1) is linear, the problem reduces to analyzing the case

$$f(x,t) = f_0(x)\cos(\omega t + \phi), \tag{3}$$

where $\omega = (2\pi/\Theta)n$ and $\phi = 0, \pi/2$. In this case the solution of (1) may be written in the form

$$u(x,t) = A(x)\cos(\omega t + \phi) + w(x,t), \tag{4}$$

where $w(x,t)$ is solution of the homogeneous equation corresponding to (1) at the same boundary conditions and the initial conditions: $w(x,0) = -A(x)\cos\phi, \dot{w}(x,0) = A(x)\omega\sin\phi$. Thus, the problem of minimizing the string response $u(x,t)$ turns to determining $A(x)$, and subsequent tailoring of $A(x)$ by cross-sectional area modulation. For $A(x)$ the following equation is