Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Wave properties in poroelastic media using a Wave Finite Element Method

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ARTICLE INFO

Article history:
Received 25 October 2013
Received in revised form
14 September 2014
Accepted 17 September 2014
Handling Editor: G. Degrande

ABSTRACT

The application of the bidirectional Wave Finite Element Method (WFE) to Biot–Allard's theory of poroelasticity is investigated. This method has been successfully used in previous elastodynamics studies. In the case of porous media, the rigidity of the layer is very low, leading to very small wavelengths, and a high dissipation rate occurs within the pores. These differences with the elastic case justify a study of their consequences on numerical results. In this paper, it is shown that despite these difficulties, the WFE provides an efficient tool to compute the waves propagating through poroelastic media. The influence of boundary conditions on wave propagation is discussed, as well as the convergence of the numerical results, depending on the spatial discretization, the order of shape functions, and the choice of the formulation. Finally, the wavenumbers predicted with this method are compared with some simplified models such as equivalent fluid models or equivalent plate models. It is shown that the WFE can be used to validate the assumptions made by the simplified models.

1. Introduction

Poroelastic materials are composed of a skeleton and of saturating fluid circulating within the pores. This results in very small density and a high level of dissipation of the energy within the pores. This is due to the combination of three dissipative mechanisms: structural damping in the frame, thermal losses between the fluid phase and the solid phase, and dissipation in the viscous boundary layers in the saturating fluid near the surface of the pores. In vibroacoustics, the saturating fluid is the ambient air, so these materials present very good sound absorbing properties. Such materials are thus commonly used in the transportation industry. Several models exist to predict their efficiency in dissipating acoustic or vibrational energy. The Biot–Allard theory [1] is often referred to as the most complete. Instead of solving the biphasic discontinuous problem at the pore scale, where the two phases are geometrically separated, this model defines the parameters of the equivalent continuous material having two phases in each point. All of the exchanges at the pore scale are taken into account in the form of volume parameters which are complex and frequency dependent. This model can be

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http://dx.doi.org/10.1016/j.jsv.2014.09.022
0022-460X/

Please cite this article as: Q. Serra, et al., Wave properties in poroelastic media using a Wave Finite Element Method, *Journal of Sound and Vibration* (2014), http://dx.doi.org/10.1016/j.jsv.2014.09.022

implemented within the framework of the Finite Element Method (FEM), but this approach leads to very large computational times for industrial configurations. A major issue today is thus the development of reduced models allowing to increase numerical efficiency without a significant loss in accuracy.

Several works deal with the reduction of the size of the problem obtained with FEM. This reduction can be done by using modal superposition techniques [2–5]. This technique is very efficient in the low frequency range, where a small number of modes capture the dynamics of the system. However, in the mid-frequency range or in the high frequency range, a large number of modes are needed, reducing the interest of the approach. Reducing the number of frequency iterations can be considered by using Padé approximants, see for example [6]. Although this method is very efficient, it lacks physical justification. The error is dependent on the order of the Padé decomposition and must be controlled by adaptive algorithms.

Alternatively models in which the reduction is applied on the exact equations can be used. Thus, the physics in the medium is simplified. Among these models, the model of localized impedance makes it possible to predict the effect of the acoustic treatment on the surrounding fluid without adding any variable for it. This effect is represented by a parameter called impedance, constant on the interface, relating the acoustic pressure and the normal acoustic velocity. Three assumptions are made in this approach. The first is that the value of the impedance is constant on the interface, the second is the value of the parameter, and the third is a localized response of the sound package. While it is very efficient and accurate in a large number of cases [7,8], such modelling can lead to significant errors when curved panels or materials with non-local response are used.

Modelling the poroelastic medium as an equivalent fluid is another possible reduction of the Biot model. This model is obtained by introducing some approximations for the motion of the solid phase, typically motionless, rigid body or limp assumptions [1,9,10]. These models assume that the solid phase displacement, the solid phase strain or the solid phase stresses are equal to zero respectively. This class of models can be used in FEM in the case of curved geometries, however their pertinence depends on the material and on the installation settings. In particular, these models should not be applied when the skeleton is excited by structural excitations, when boundary conditions have a large influence on the dynamic behaviour of the panel [9], or when the parameters of the porous materials are such that the two phases are strongly coupled.

Finally, it is worth noting that some papers deal with the modelling of poroelastic plates as equivalent plates, using *a priori* physical assumptions to derive an expression of an equivalent plate based for example on Kirchhoff–Love's theory or high-order theories [11–15].

Another class of methods has emerged to reduce the computational cost in the mid-frequency range and in the high-frequency range. These approaches are based on the waves propagating spatially in the medium. Among them is the Wave Based method [16–18] in particular, in which the exact solution is approximated by a superposition of wave-based shape functions satisfying exactly the volume equations but approximatively for the boundary conditions. This method leads to very fast convergence for convex geometries, and has been applied to acoustic cavities and poroelastic layers. However the main difficulty of this approach lies in its application to complex geometries.

The Transfer Matrix Method [19,1] (TMM) is a famous semi-analytical method. It can be applied to arbitrary multilayer configurations, at any frequency, and has very high computational efficiency. The method considers a flat infinite panel excited by an incident pressure field. The analytical expression of the transfer matrix relating the two faces of the panel is derived based on the wavenumbers propagating in the medium. Numerical resolution makes it possible to compute acoustic indicators such as the impedance coefficient or the Transmission Loss. However, the assumptions of the method may lead to large errors, especially in the low-frequency range or when reduced dimensions are considered.

Finally, wave propagation has also been studied in elastic media such as rails or beams using periodic structures theory [20] and Floquet theory. The Wave Finite Element Method (WFE) [21,22] has been derived to apply to complex geometries. From the knowledge of FE matrices, a transfer matrix relating the two state vectors at the two extremities of an unidirectional waveguide is derived. It has been successfully applied in the case of elastic or viscoelastic media, fluid-filled cylinders [23], ribbed plates [24], multilayer configurations [25], used for damage detection [26], investigation of coupling effects between two coupled parallel waveguides [27] or model reduction in the case of deterministic or stochastic structures [28–30]. Finally, the method has also been extended to bidirectional wave propagation and applied to study the effects of damping on wave propagation in plates [31–35].

However, to the authors' knowledge, no other contribution deals with the application of this method to poroelastic domains. The potential interest of such a method, in the case of poroelastic media, lies in several points. Firstly, the FE modelling of Biot–Allard's theory makes it possible to model all of the energetic exchanges between the two phases, contrary to equivalent fluid models or equivalent plate models. Secondly, and contrary to the localized impedance model, it allows geometry effects to be dealt with. Thirdly, if wavenumbers predicted by the present approach can be considered as exact, they can be used to validate models with simplified physics such as equivalent plate theories or equivalent fluid models, and also to determine acoustic parameters of acoustic treatments (see for example [33,36] for the case of sandwich viscoelastic panels). Finally, the method extends to model reduction.

This paper presents a study of applicability of the WFE to poroelastic media. It aims at validating the approach in terms of convergence of the results and influence of the formulations. Indeed, contrary to elastic or viscoelastic media where the primal displacement formulation is typically used for vibration problems, the choice between displacement–displacement formulations or displacement–pressure formulations can be discussed. The displacement–pressure formulation is used more often than the displacement–displacement formulation and leads to the same results in frequency domain in FEM