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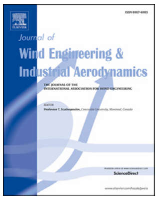

# A new Gaussian-based analytical wake model for wind turbines considering ambient turbulence intensities and thrust coefficient effects

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ABSTRACT

A new wake model for wind turbines considering ambient turbulence intensity and thrust coefficient effects is proposed by numerical and analytical studies. Firstly, two kinds of operating condition with different thrust coefficients under two types of inflow with different ambient turbulence intensity are simulated for a model and a utility-scale wind turbine by using large eddy simulation (LES). The predicted mean velocity and turbulence intensity in the wakes of two wind turbines are compared with those obtained from the wind tunnel tests to validate numerical models. Subsequently, a new wake model is proposed to predict the mean velocity and turbulence intensity distribution in the wake regions of wind turbines. The model is derived based on the axial symmetry and self-similarity assumption for wake deficit and added turbulence intensity. All parameters of the proposed model are determined as the function of ambient turbulence intensity and thrust coefficient identified based on the various large eddy simulations. The velocity deficit and added turbulence intensity in the wake predicted by the new wake model show good agreement with the LES simulations and experimental results in the near and far wake regions.

## 1. Introduction

Wind turbines in a wind farm operating in the downwind wake flow are subjected to two main problems: decreased energy production due to the velocity deficit and increased fatigue loading due to the added turbulence intensity generated by the upwind turbine (Vermeer et al., 2003; Barthelmie et al., 2009). Especially in the offshore wind farm, where the ambient turbulence intensity is lower than that in the terrestrial boundary layer, the wakes recover more slowly and bring severer effects (Ishihara et al., 2004; Wu and Porté-Agel, 2012). Therefore, an accurate evaluation of the wake effect is essential in the wind farm layout design in order to improve the power efficiency and the lifetime of the turbine.

Prediction of wake effect requires detailed understanding of the behavior of wake flow and its interaction with atmospheric boundary layer. In previous studies, the wake characteristics in the atmospheric boundary layer have been investigated by wind tunnel tests (Ishihara et al., 2004; Chamorro and Porte-Agel, 2009), however it is difficult to capture the detailed turbulence structure due to the constraint of measurement.

In recent years, computational fluid dynamics (CFD) has been used to study wind turbine wake flow and to optimize wind farm layout (Sanderse et al., 2011). In these studies, the large-eddy simulation (LES) was popularly used for the study of wind turbine wake characteristics, and the wind turbine induced forces were modelled using either of the two approaches, the generalized actuator disk model (ADM) or actuator line model (ALM). The ALM is used to reproduce detailed three-dimensional rotational effects, like tip vortices. However, it is noted that finer mesh and smaller time steps are required for ALM, thus this method is costly for LES simulation of a large wind farm. According to the study in reference (Witha et al., 2014), the CPU time of ALM simulation is greatly enhanced compared to the ADM simulation by a factor of 4–12 depending on the grid resolution. The earliest version of ADM is the actuator disk model without rotation (ADM-NR), in which the turbine induced force is parameterized as an overall thrust force uniformly acting on the rotor disk (Jiménez et al., 2007; Calaf et al., 2010; Goit and Meyers, 2015). Another extended ADM uses the blade element momentum (BEM) theory (Burton et al., 2011) to calculate the lift and drag forces and then unevenly distribute them on the actuator disk. This modified approach is referred to as the actuator disk model with rotation (ADM-R). The ADM-R has been employed in LES simulation and validated by the wind tunnel tests for the wind turbine wakes in turbulent boundary layers (Wu and Porté-Agel, 2011, 2012). Although the detailed

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characteristics of mean velocity and turbulence in the turbine wake have been examined in these studies (Wu and Porté-Agel, 2011; Xie and Archer, 2014), the effect of ambient turbulence intensity and thrust coefficient require further systematic investigation.

In comparison to wind tunnel tests and numerical simulations, wake models have advantages in designing and optimizing wind farm layout because of its simplicity and high efficiency (Crespo et al., 1999). The wake can be generally divided into near wake region and far wake region (Vermeer et al., 2003). The near wake region typically has a length less than three diameters downwind the turbine (Crespo and Hernández, 1996), which is complicated to cope with due to the fact that it is significantly affected by the blade aerodynamics, stalled flow, tip vortices as well as nacelle and tower (Vermeer et al., 2003; Xie and Archer, 2014). Thus, most wake modelling mainly focus on the far wake region, where the wake is fully developed and the velocity deficit and the added turbulence intensity can be assumed axisymmetric and have self-similar distributions in the wake cross-sections (Vermeer et al., 2003).

Prediction of the velocity deficit is the primary objective of wake models. One of the classical and widely used wake model for velocity deficit was proposed by Jensen (1983), and was developed further by Katic et al. (1986), which assumes a linearly expanding wake with a uniform profile, termed “a top-hat shape”, for the velocity deficit. The Katic et al.'s model only considers the mass conservation. More recently Frandsen (Frandsen et al., 2006) proposed a model that applied the balance of momentum in addition to the mass conservation. It still took a top-hat assumption for the velocity deficit. In comparison with the top-hat assumption, Gaussian distribution is more reasonable for the velocity deficit profile in wake cross section, which was derived by Ishihara et al. (2004) and observed in the experimental data (Ishihara et al., 2004; Chamorro and Porte-Agel, 2009) and numerical simulations (Xie and Archer, 2014). It was also employed in several wake models (Ishihara et al., 2004; Ainslie, 1988; Bastankhah and Porté-Agel, 2014; Gao et al., 2016). However, there are still problems of robustness and universality for these models.

Modelling the turbulence in wind turbine wake flows is also important since the wake induced turbulence increases the fatigue loading of the downwind turbine. Considering the complex nature of turbulence, it is common to model the maximum added turbulence intensity $\Delta I_{1\max}$ , which normally occurs at the top tip height level. Based on the measurement data, Quarton and Ainslie (1989) proposed a widely used empirical expression for $\Delta I_{1\max}$, which is proportional to thrust coefficient and ambient turbulence intensity. The distance from wind turbine was normalized by the estimated near wake length $x_N$ defined by Vermeulen (1980). The parameters in Quarton's model were modified by Hassan (Hassan et al., 1990) based on wind tunnel measurements. Crespo and Hernández (1996) proposed a similar model for $\Delta I_{1\max}$, which is related to the induction factor and ambient turbulence intensity. These three wake models are quite similar and show overestimation in the near wake region. In addition, the distribution of added turbulence intensity in the wake cross section is also important in the wind farm layout design and has not been investigated yet.

In this paper, a new wake model for wind turbines is proposed to consider the ambient turbulence intensity and thrust coefficient effects by numerical and analytical studies. In section 2, the numerical model used in this study is described and the systematic simulations are conducted and compared with the experimental data. Section 3 presents a new wake model proposed in this study. The accuracy of the proposed model and the conventional wake models are examined by the LES simulations and the wind tunnel tests. Finally, conclusions of this study are summarized in section 4.

## 2. Numerical model and validation

In this study, LES is employed to simulate the wind turbine wake flows, and the governing equations are presented in section 2.1. The accuracy of the wind turbine model is validated in section 2.2 by comparing the calculated thrust coefficients with the measured data of a model and a utility-scale wind turbine, respectively. Section 2.3 describes the set-up of the numerical simulations, including the computational domain and the main parameters used in each case. The turbulent inflows generated in the numerical wind tunnel are validated in section 2.4. Finally, the characteristics of the mean velocity and turbulence intensity in the wake region under different conditions are investigated and validated by the wind tunnel test in section 2.5.

### 2.1. Governing equations

In the LES, large eddies are directly computed, while the influences of eddies smaller than grid spacing are parameterized. The Boussinesq hypothesis is employed, and the Smagorinsky-Lill model (Smagorinsky, 1963) is used to calculate the subgrid-scale (SGS) Reynolds stress.

The governing equations are filtered incompressible Navier-Stokes equations and are expressed as (Ferziger and Perić, 2002):

$$\frac{\partial \rho \tilde{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \tilde{u}_i) + \frac{\partial}{\partial x_j}(\rho \tilde{u}_i \tilde{u}_j) = \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \tilde{u}_i}{\partial x_j}\right) - \frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + f_i \tag{2}$$

where $\tilde{u}_i$ and $\tilde{p}$ are respectively filtered velocities and pressure, $\rho$ is the air density, $\mu$ is the dynamic viscosity, $f_i$ is the source term to present the external force corresponding to the effects of the wind turbine on the momentum, and $\tau_{ij}$ is the SGS stress, which is modelled as:

$$\tau_{ij} = -2\mu_t \tilde{S}_{ij} + \frac{1}{3}\tau_{kk}\delta_{ij} \tag{3}$$

where $\mu_t$ denotes SGS turbulent viscosity, $\delta_{ij}$ is the Kronecker delta and $\tilde{S}_{ij}$ is the rate-of-strain tensor for the resolved scale which is defined as follows:

$$\tilde{S}_{ij} \equiv \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right) \tag{4}$$

Smagorinsky-Lilly model is used for the SGS turbulent viscosity, i.e.:

$$\mu_t = \rho L_S^2 |\tilde{S}| = \rho L_S^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \tag{5}$$

where $L_S$ denotes the mixing length for subgrid-scales and it is calculated by:

$$L_S = \min\left(\kappa\delta, C_S V^{1/3}\right) \tag{6}$$

$\kappa$ is the von Karman constant, 0.42, $\delta$ is the distance to the closet wall and $V$ is the volume of a computational cell. In this study, Smagorinsky constant $C_S$ is determined as 0.032 based on the study performed by Oka and Ishihara (2009).

For the wall-adjacent cells, the wall shear stresses are obtained from the laminar stress-strain relationship in laminar sublayer:

$$\frac{\tilde{\mu}}{\mu_\tau} = \frac{\rho \mu_\tau y}{\mu} \tag{7}$$

Provided that the mesh cannot resolve the laminar sublayer, the centroid of the wall-adjacent cells is assumed to fall within the logarithmic region of the boundary layer, and then the law of the wall is employed as follows:

$$\frac{\tilde{\mu}}{\mu_\tau} = \frac{1}{\kappa}\ln E\left(\frac{\rho \mu_\tau y}{\mu}\right) \tag{8}$$

where $\tilde{\mu}$ is the filtered velocity tangential to the wall, $y$ is the distance between the center of the cell and the wall, $\mu_\tau$ is friction velocity, and the