Journal of Wind Engineering & Industrial Aerodynamics 176 (2018) 273–289

Contents lists available at ScienceDirect

Journal of Wind Engineering & Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia

Short communication

3D numerical analysis on wind and rain induced oscillations of water film on cable surface

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ARTICLE INFO

Keywords:
Three-dimensional
Rain–wind induced vibration
Stay cable
Rivulet
Finite difference method
Axial flow of water film

ABSTRACT

Rain and wind induced vibration (RWIV) of cable on cable-stayed bridges is a three-dimensional phenomenon. However, previous studies about the numerical analysis on the RWIV mostly concentrate in 2D model, because of the complexity of Computational Fluid Dynamics (CFD). In this paper, a 3D equation of water film evolution is established for the first time, based on the lubrication theory, in which the effects of gravity, water film surface tension, aerodynamic lift and the vibration of the cable are considered. By solving the equations using Finite Difference Method (FDM), the water film thickness variation time history curves at every point on the cable surface are obtained. The results show that an upper rivulet forms throughout the cable along with the axial direction periodically and the phenomenon of axial flow of water film is very obvious. The locations of the upper rivulet are different at every section along with the axial direction at the same moment. The dominant frequency of the evolution of water film is close to the nature frequency of cable.

1. Introduction

Rain–wind induced vibration (RWIV) is a large amplitude and low frequency vibration of cables of cable-stayed bridges and suspenders of arch bridges under wind and rain (Bosdogianni and Olivari, 1996). This phenomenon was first reported on the Meikonishi Bridge by Hikami and Shiraishi (1988), and researchers repeatedly observed the same phenomenon worldwide in other bridges (Matsumoto et al., 1990; Main and Jones, 1999; Chen et al., 2003; Ni et al., 2007; Zuo et al., 2008). To investigate the nature of RWIV, a series of wind tunnel tests (Matsumoto et al., 1990, 1995; Flamand, 1995; Gu et al., 2002; Cosentino et al., 2003; Gu, 2009; Gu and Du, 2005; Li et al., 2010a,b; Xu et al., 2011; Du et al., 2013; Li, 2015) were performed. Based on the results of the tests, the basic characteristics of RWIV are as follows: (1) RWIV occurs over a certain range of wind speeds along with light or moderate rain; (2) upper and lower rivulets are formed on the cable surface and oscillate with low order modes; and (3) the vibration amplitude is related to the length, the inclination direction, the surface material of the cable, the wind yaw angle and the damping. To investigate the nature of RWIV, three main types of theoretical models have developed by researchers:

(1) The rivulets are simulated as moving particles on the cable surface. The aerodynamic coefficients of the cable are substituted into the motion equations of the cable and rivulets, considered to be known parameters that are obtained by force or pressure tests of the cable section model with artificial rivulets in a wind tunnel (Yamaguchi, 1990; Peil and Nahrath, 2003; Seidel and Dinkler, 2006; Li et al., 2009).
(2) The motion equations of rivulets are not established, and the forces on the cable caused by rivulet motion, which are considered to be known parameters based on the assumption of the rivulet motion law, are substituted into the cable motion equation (Xu and Wang, 2003; Li et al., 2007; Bi et al., 2010; Li et al., 2010a,b).
(3) Lubrication theory is used to simulate the formations and oscillations of rivulets by assuming that there is a continuous water film on the cable surface. Lemaitre et al. (2007) developed a two-dimensional model based on lubrication theory that describes the evolution of a thin film subject to gravity, surface tension, wind and motion of the cylinder. They added wind as an exterior load expressed as the pressure and friction coefficients$C_p(\theta)$ and $C_f(\theta)$ (Reisfeld and Bankoff, 1992) to simulate the formations of

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https://doi.org/10.1016/j.jweia.2018.03.026
Received 21 March 2017; Received in revised form 7 February 2018; Accepted 24 March 2018
Available online 11 April 2018

rivulets and to study the variation of water film around the cable. To consider the effect of cable movement on the water film, Xu et al. (2011) modified the motion equation of Lemaitre's model by assuming that the dynamic characteristics of the cable were known conditions, and they investigated the evolution of a water film subject to gravity, wind pressure, friction and surface tension. To obtain the variation of the water film around cable with time, Taylor and Robertson (2011) modified Lemaitre's model by the substitution of wind pressure and friction coefficients $C_p(\theta,t)$and $C_f(\theta,t)$, which vary with time through numerical calculation. Bi et al. (2013) established the 2D coupled equations for water film evolution and cable vibration for the first time based on Xu's and Taylor's models combining with vibration theory of a single-mode system. The wind pressure coefficient $C_p(\theta,t)$and friction coefficient $C_f(\theta,t)$with different water-film morphologies are obtained by the CFD software Fluent 6.3.

It can be seen that the previous studies based on the lubrication theory are all about the two-dimensional model. But actually, RWIV is a very complex three-dimensional phenomenon. In this paper, based on the lubrication theory, a 3D evolution equation of water film on the cable surface induced by wind and rain is established for the first time. Besides, the finite difference scheme of the equation is given. Based on the experiment of Li et al. (2010a,b), the parameters are input into the model. Drawing lessons from Xu et al. (2011), the dynamic characteristics of the cable are regarded as known conditions. According to Robertosn et al. (2010) and Xu et al. (2011), the effects of aerodynamic force are simplified. The wind pressure coefficient $C_p(\theta)$ and friction coefficient $C_{f\theta}(\theta)$ of a dry cable with circular cross-section are applied while another friction coefficient C_{fz} is ignored in this paper. By solving the equation, the thicknesses of the water film at every computational point are obtained. The fast Fourier transform is performed on the time history curves, and the frequencies of the thicknesses of water film are obtained. In order to discuss the 3D effect of axial flow of water film, the thickness of the water film on adjacent two sections are analyzed. In addition, the relationship between cable vibration and evolution of water film are analyzed.

2. Model

2.1. Evolution equation of water film

As shown in Fig. 1(a), the cable with a radius of R_c and a horizontal inclination angle of $\alpha(0^\circ < \alpha \leq 90^\circ)$ is acted upon by gravity gand horizontal wind with the speed of U. The wind yaw angle is $\beta(0^\circ \leq \beta \leq 90^\circ)$ and the attack angle is 0°. The gravity is divided into the component in the A-A cross-section g_N and the component along the axial direction of cable g_Z.

$$g_N = g\cos\alpha \tag{1a}$$

$$g_Z = g\sin\alpha \tag{1b}$$

Like the gravity, the airflow effect is divided into two components U_N and U_Z, too.

$$U_N = \frac{U\cos\beta}{\cos\delta} \tag{2a}$$

$$U_Z = \sqrt{U^2 - U_N^2} \tag{2b}$$

where,

$$\delta = \arctan(\sin\alpha\cdot\tan\beta) \tag{3}$$

The angle between g_N and U_N is ψ, where, $\psi = \delta + \frac{\pi}{2}$.

The cylindrical coordinate system$(\mathbf{e_r},\mathbf{e_\theta},\mathbf{z})$ is adopted, as shown in Fig. 1(b). The A-A cross-section of the cable given in Fig. 1(a) is taken as the research object. The forces to which the water film is subject are listed in Fig. 1(c). g_N^θ and g_N^r are the components of g_N in two directions of θ and r respectively. The angle between g_N and g_N^r is φ.

Based on the lubrication theory, supposing that there is continuous water film on the surface of the stay cable. The coordinate of any point in the water film is $(r,\theta,z)R_c \leq r \leq R_c + h$, where h is the thickness of water film.

The equation of the water film evolution is derived from the three-dimensional Navier-Stokes equations:

$$\begin{cases} \rho\dfrac{D\mathbf{u}}{Dt} = \rho(\mathbf{g} + \ddot{\mathbf{y}}) - \nabla p + \mu\Delta\mathbf{u} \\ \nabla\cdot\mathbf{u} = 0 \end{cases} \tag{4}$$

where $\mathbf{u}$ is the velocity of the water film, ρ is the density of the water, p is the pressure in the water film, μ is the kinematic viscosity coefficient of the water, $\ddot{\mathbf{y}}$ is the acceleration of cable.

Velocity$\mathbf{u}$ is expressed as the component form$\mathbf{u} = u_r\mathbf{e_r} + u_\theta\mathbf{e_\theta} + u_z\mathbf{e_z}$, then the Navier-Stokes equations can be written as:

$$\begin{aligned} &\rho\left(\partial_t u_r + u_r\partial_r u_r + \frac{u_\theta}{r}\partial_\theta u_r + u_z\partial_z u_r - \frac{u_\theta^2}{r}\right) \\ &= \rho(\mathbf{g}\cdot\mathbf{e}_r - \ddot{y}\cdot\mathbf{e}_y\cdot\mathbf{e}_r) - \partial_r p + \mu\left\{\partial_r\left[\frac{1}{r}\partial_r(ru_r)\right] + \frac{\partial_\theta^2 u_r}{r^2} + \partial_z^2 u_r - \frac{2\partial_\theta u_\theta}{r^2}\right\} \end{aligned} \tag{5a}$$

$$\begin{aligned} &\rho\left(\partial_t u_\theta + u_r\partial_r u_\theta + \frac{u_\theta}{r}\partial_\theta u_\theta + u_z\partial_z u_\theta + \frac{\partial_r u_\theta}{r}\right) \\ &= \rho(\mathbf{g}\cdot\mathbf{e_\theta} - \ddot{y}\cdot\mathbf{e}_y\cdot\mathbf{e_\theta}) - \partial_\theta p + \mu\left\{\partial_r\left[\frac{1}{r}\partial_r(ru_\theta)\right] + \frac{\partial_\theta^2 u_\theta}{r^2} + \partial_z^2 u_\theta + \frac{2\partial_\theta u_r}{r^2}\right\} \end{aligned} \tag{5b}$$

$$\begin{aligned} &\rho\left(\partial_t u_z + u_r\partial_r u_z + \frac{u_\theta}{r}\partial_\theta u_z + u_z\partial_z u_z\right) \\ &= \rho(\mathbf{g}\cdot\mathbf{e_z} - \ddot{y}\cdot\mathbf{e}_y\cdot\mathbf{e_z}) - \partial_z p + \mu\left[\frac{1}{r}\partial_r(r\cdot\partial_r u_z) + \frac{\partial_\theta^2 u_z}{r^2} + \partial_z^2 u_z\right] \end{aligned} \tag{5c}$$

$$\frac{1}{r}\partial_r(ru_r) + \frac{1}{r}\partial_\theta u_\theta + \partial_z u_z = 0 \tag{5d}$$

Because the thickness of the water film is very thin and the velocity of the water film is very small, the Reynolds number of the water film is very small. Calculation verified that the Reynolds number of water film is about 1. So the assumption that the Reynolds number of the water film $\mathrm{Re}_h = hu/\nu \approx 1$ applied in Lemaitre et al. (2007) is still used in this paper. ν is the kinematic viscosity of water film, so Eqs. (5a), (5b) and (5c) can be written as

$$\partial_r p = \mu\left\{\partial_r\left[\frac{1}{r}\partial_r(ru_r)\right] + \frac{\partial_\theta^2 u_r}{r^2} + \partial_z^2 u_r - \frac{2\partial_\theta u_\theta}{r^2}\right\} - \rho[g_N\sin(\theta-\delta) - \ddot{y}\sin\theta] \tag{6a}$$

$$\partial_\theta p = \mu\left\{\partial_r\left[\frac{1}{r}\partial_r(ru_\theta)\right] + \frac{\partial_\theta^2 u_\theta}{r^2} + \partial_z^2 u_\theta + \frac{2\partial_\theta u_r}{r^2}\right\} - [\rho g_N\cos(\theta-\delta) - \ddot{y}\cos\theta]r \tag{6b}$$

$$\partial_z p = \mu\left[\frac{1}{r}\partial_r(r\cdot\partial_r u_z) + \frac{\partial_\theta^2 u_z}{r^2} + \partial_z^2 u_z\right] - \rho g_z \tag{6c}$$

The boundary conditions of the equation include:

(1) Displacement boundary conditions

Water is relatively static to the surface of the stay cable at the bottom of the water film.

$$u_r|_{r=R_c} = u_\theta|_{r=R_c} = u_z|_{r=R_c} = 0 \tag{7}$$