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Reduced order modeling of two degree-of-freedom vortex induced vibrations of a circular cylinder

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ABSTRACT

The effect of coupled transverse and in-line motion of an elastically mounted rigid circular cylinder, subjected to vortex induced vibrations (VIV), is predicted using a reduced-order model. The model comprises of coupled wake and structural oscillators, where the nonlinearities in the fluid damping and forcing terms of the structural oscillator are retained. The classical van der Pol equation is used to model the wake oscillator. The unknown model constants are tuned to fit to experimental data. The influence of these tuning constants on the model performance are identified. The nonlinear contributions are shown to be insignificant in predicting the VIV characteristics associated with the transverse (y-only) oscillations of the cylinder at low Re. Surprisingly, the nonlinear terms were found to play a key role in predicting the two degree-of-freedom (2 DoF) motion of the cylinder. The model results for the cylinder with mass ratios in the low and moderate ranges are in good agreement with the experiments.

1. Introduction

Alternate shedding of vortices behind a circular cylinder causes oscillatory forces on it. When the frequency of vortex shedding is close to one of the natural frequencies of the structural system, the bluff body tends to vibrate and its amplitude is significantly enhanced in the synchronization ranges. This is termed Vortex Induced Vibrations (VIV), which indeed is a two-way coupled Fluid Structure Interaction (FSI) phenomenon. In reality, the structural vibration alters the surrounding flow field which in turn modifies the forces induced on the cylinder. The occurrence of VIV can be observed in flow over marine cables, offshore pipelines, risers etc. In the literature, investigations on VIV have mainly focussed on drag reduction through vortex shedding suppression (Huang, 2011; Bearman and Owen, 1998; Patil et al., 2011; Sarwar and Ishihara, 2010; Muddada and Patnaik, 2010) and few attempts on harvesting energy (Bernitsas et al., 2008; Antonio et al., 2012; Dhanwani et al., 2013). Since most of these applications have a circular cross section and a cylindrical geometry, VIV of circular cylinder has been extensively studied (see Kawai, 1993; Khalak and Williamson, 1997, 1999; Blackburn et al., 2001; Sarpkaya, 2004; Williamson and Govardhan, 2004).

Experiments on VIV of long slender cylinders are relatively few, owing to the need for large facilities and relatively complicated instrumentation. Alternately, for the range of turbulent Reynolds numbers that occur in most practical applications, numerically solving the equations of structural motion coupled to fully non-linear Navier-Stokes equations involve high computational cost and difficulties in implementation. In the review of Wu et al. (2012), detailed discussions of recent experiments and computational works on VIV of long slender structures can be found. Such complexities in the setting up of experiments and computational limits regarding full scale numerical simulations have paved the way for alternate reduced-order semi-empirical models. These VIV models are reviewed by Gabbai and Benaroya (2005).

Lower order models typically employ a separate mathematical equation to represent the wake, and a structural oscillator, formulating the dynamics of the vibrating structure as well as its coupling. An early model of Bishop and Hassan (1964), modelled the wake dynamics using a single wake variable governed by a van der Pol equation. For a detailed study on the nature of wake oscillator model see Ref. Xu et al. (2015). Balasubramanian and Skop (1996), used wake oscillators (diffusive van der Pol equation) that are continuously distributed along the span of a slender structure, thereby describing 3-dimensional spanwise features of vortex shedding. In the subsequent improvements (Skop and Balasubramanian, 1997; Plaschko, 2000; Skop and Luo, 2001), the van der Pol oscillator was further modified for 3-dimensional flows past structures having high aspect ratio. Larsen (1995) introduced a single DoF model for VIV of light and flexible structures based on generalization of van der

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Pol type forcing function. Other categories of low-order models include the Voltera theory based model for the application of VIV of large-span bridges (Xu et al., 2017), the nonlinear oscillator based on circle map (Olinger, 1998), the force-decomposition models originally based on the work of Sarpkaya (1978), and the recent Proper Orthogonal Decomposition (POD) based methods, (Lieu et al., 2006; Liberge and Hamdouni, 2010), applied to general fluid-structure interaction problems. The recent 2 DoF circular cylinder model, based on linear theory, proposed by Dhanwani et al. (2013), was used in designing an energy harvester.

Historically, two-dimensional uniform flow over an elastically mounted circular cylinder has been extensively chosen as a representative model for studying VIV (Khalak and Williamson, 1996, 1999; Govardhan and Williamson, 2000; Dahl et al., 2010). The relevance of such a paradigm to the actual 3-dimensional complex fluid flow systems was highlighted by Williamson and Govardhan (2008). In most VIV studies of circular cylinders, the effect of in-line motion was assumed to be insignificant, with a focus on the transverse vibrating (y-only) motion. Relatively recent experiments (Jauvtis and Williamson, 2004; Sanchis et al., 2008) have shown the significance of in-line motion in understanding VIV of cylinders with low mass ratios (ratio of cylinder mass to displaced fluid mass). According to Jauvtis and Williamson (2004), the effect of such a coupled in-line and transverse motion results in a newly discovered triplet (2T) mode of vortex shedding behind the cylinder. A new super upper branch in the cylinder amplitude response was observed for low mass ratios.

Facchinetti et al. (2004) have classified the low-order models for VIV of an elastically mounted cylinder, freely oscillating in the transverse direction when subjected to uniform 2-dimensional flow, based on the type of coupling between the wake dynamics and the structural motion. The three types of linear coupling, namely displacement, velocity and acceleration, formulate the displacement, velocity or acceleration of the cylinder, respectively, to be the forcing term in the wake oscillator. Facchinetti et al. (2004) inferred based on linear theory that inertial or acceleration coupling is the most effective coupling strategy to predict both qualitative and quantitative features of 2-dimensional VIV phenomenon.

The major focus of all the reduced order models of VIV has been to capture the amplification of cylinder motion in the synchronization regime and the lock-in range of frequencies. To the authors' knowledge, there is no single reduced order model in the literature that quantitatively predicts these features over a wide range of mass ratios for 2 degree of freedom (DoF) motion of circular cylinder for different flow Reynolds numbers. In the present work, a 2 DoF (both in-line and transverse vibrations of the cylinder) model is proposed by including certain nonlinearities in the structural model. The model is tuned to match the experimental results of Jauvtis and Williamson (2004) and to capture the effect of 2 DoF cylinder vibrations in different ranges of mass ratios. The influence of nonlinear terms in the wake oscillator model is analyzed. The nonlinear contributions are found to be relatively insignificant in the low Re regimes ($Re < 300$) compared to flow regimes at higher Re. The coupling between the two oscillators, under pure transverse and combined motion are analyzed for the correlation effects.

This article is organized into five sections. Section 2 presents the 2 DoF model formulation and the solution methodology. In Section 3, validations of the model, against pure transverse cylinder oscillations and pure in-line vibration experiments, and the behaviour of the tuning coefficients are discussed. The numerical predictions of the model with application to 2 DoF cylinder motion and the significance of nonlinear contributions are discussed in Section 4. Concluding remarks are given in Section 5.

2. Model formulation

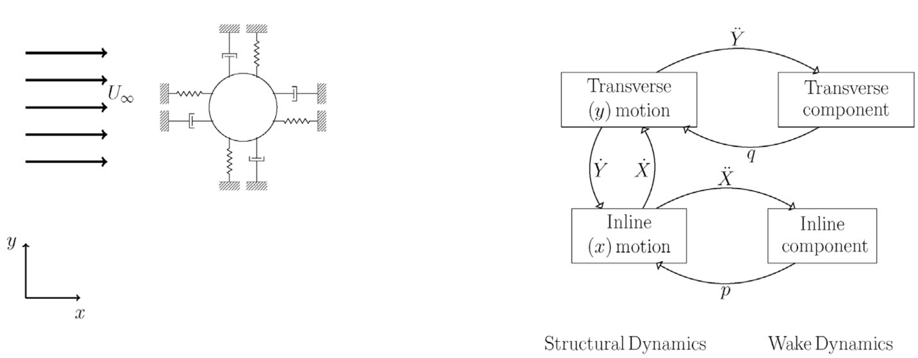
A simplified model of circular cylinder cross-section free to oscillate in transverse and stream-wise direction subjected to uniform flow is shown in Fig. 1a. The rigid cylinder is elastically mounted with effective stiffness k_x, k_y and damping ratios ζ_x, ζ_y along x and y directions respectively (Dhanwani et al., 2013). In the present study, dynamics of the cylinder wake is modelled using a reduced order wake oscillator, while the structural oscillator captures cylinder motion. The coupling between the two oscillators and the x and y motion is pictorially depicted in Fig. 1b.

The coupled equations governing the structural oscillator and the cylinder motion in transverse and stream-wise directions, (Dhanwani et al., 2013), are given as

$$\ddot{Y} + \left(2\zeta_y\delta_y + \frac{\gamma}{\mu}\right)\dot{Y} + \left(2\frac{\chi}{\mu}\right)\dot{X} + \delta_y^2 Y = Nq \tag{1}$$

$$\ddot{X} + \left(2\zeta_x\delta_x + 2\frac{\gamma}{\mu}\right)\dot{X} - \left(\frac{\chi}{\mu}\right)\dot{Y} + \delta_x^2 X = Mp \tag{2}$$

where the $(\cdot)$ represents the time derivatives. The non-dimensional parameters used in the above equations are defined in Table 1. U_∞ is the free stream velocity, ν is fluid kinematic viscosity, D is the cylinder diameter, L is the span-wise length of cylinder, f_X and f_Y are the frequencies of cylinder motion in the x and y directions respectively, and the added mass coefficient $C_A = 1$ for a circular cylinder. The wake variables q, p represent, qualitatively, the normalized lift and drag force coefficients respectively (Ogink and Metrikine, 2010). The model includes asymmetric fluid-damping like terms for $\dot{X}$ in Eqn. (2) and for $\dot{Y}$ in Eqn. (1). In the 2 DoF model (Dhanwani et al., 2013), the coefficient of these fluid-damping like terms is taken as $\frac{\chi}{\mu} = 0$ for the adopted analytical method. The experiments of Jauvtis and Williamson (2004) suggest that the effect of coupled X and Y motions cannot be underestimated, particularly for the cases of cylinder motion with low mass ratios ($\mu < 6$). The fluid-damping coefficients and the parameters M, N are defined as $\gamma = \frac{C_{D0}}{4\pi St} g(t)$, $\chi = \frac{C_{L0}}{4\pi St} g(t)$, $M = \frac{C_{D0}+1}{(16\pi^2 St^2 \mu)} g(t)$, $N = \frac{C_{L0}}{(16\pi^2 St^2 \mu)} g(t)$ where C_{D0},



(a) (b)

Fig. 1. (a) Cross-sectional view of an elastically mounted rigid circular cylinder subjected to uniform flow (b) schematic of coupled 2 DoF motion in the present model.