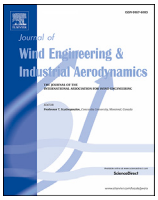
Journal of Wind Engineering & Industrial Aerodynamics 174 (2018) 382–390

Contents lists available at ScienceDirect

Journal of Wind Engineering & Industrial Aerodynamics

journal homepage: www.elsevier.com/locate/jweia



A numerical wake alignment method for horizontal axis wind turbines with the lifting line theory

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ARTICLE INFO

Keywords:
Horizontal axis wind turbine
Lifting line theory
Wake alignment
Analysis method

ABSTRACT

A method to analyze the flow around horizontal axis wind turbines based on the lifting line theory is presented. A non-linear vortex wake alignment scheme is proposed. The wake is discretized in straight line vortices. The velocity induced by the vortex system is computed in multiple sections downstream of the rotor plane. The effect of the radial velocity component is neglected and, therefore, the orientation of each straight line vortex is only given by the axial and tangential velocity components. The convergence of the numerical integration with increasing wake discretization is studied and results are compared with the analytical solution. The effect of increasing number of alignment sections is also analyzed. The results are compared to the ones computed with the constant pitch wake, aligned at the lifting lines. An increase of the blades circulation and power coefficient is obtained. Far downstream the wake geometry does not significantly vary along the axial direction. The numerical results are compared to experimental data available from the literature.

1. Introduction

The use of windmills to harvest energy from the wind is quite old. However, in the last decades the interest in wind energy has grown tremendously due to the increasing need to incorporate renewable sources in the energy mix for electrical power production and the fast technological development in the wind energy industry. As part of this technological development, the horizontal axis wind turbine has become the most used mechanical device for wind energy harvesting. Its aerodynamic modeling is of the utmost importance in turbine design and, therefore, has become a subject of intense research.

The complexity of the flow around wind turbines makes its modeling challenging and simplifications are naturally made. The models may still be analytically and numerically complex, even if one neglects viscosity and assumes steady state conditions.

In the present work we approach the aerodynamic modeling of a turbine rotor with the lifting line theory. This theory, originally introduced by Lanchester and Prandtl for wings, was applied to propellers in the classical works of Betz (1919) and Goldstein (1929). In these classical works the linear theory was applied to the lifting line model of the rotor blades. These were represented by radial bound vortices of varying circulation that shed free vortices lying on perfect helical surfaces of constant pitch. It was shown by Betz (1919) that this free vortex configuration, moving axially as a rigid surface, will minimize the kinetic energy losses for a given thrust loading and, hence, this theory is strictly valid for the so-called optimum propellers. By solving the corresponding potential flow problem Goldstein (1929) determined the optimum circulation distribution for a hubless propeller in a uniform flow.

This work was extended by Lerbs (1952) to moderately loaded propellers of arbitrary circulation distribution (non-optimum propellers) using the induction factor concept proposed by Kawada (1933). In this model the helical trailing vortices are of constant pitch in the axial direction. However, they lie on helical vortex sheets that no longer have a constant radial distribution of pitch. The crucial assumption departing from strict linear theory, enabling the application to moderately loaded propellers, was that the pitch of the vortex helical lines should be determined by aligning the trailing vortices with the local fluid velocity at the lifting lines.

In a fully non-linear inviscid theory, the vortex sheet strength vector should be aligned with the local velocity field. It is well known that under the induced velocity field the vortex sheets deform and tend to roll up at the blade tips. For a rotor this also implies that the vortex wake contracts in the case of a propeller and expands in the case of a turbine. The inviscid modeling of the vortex sheet roll-up process requires special numerical care and may be disregarded in many cases, such as for the purpose of power and thrust predictions. In such cases, a partially aligned

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https://doi.org/10.1016/j.jweia.2018.01.028
Received 30 August 2017; Received in revised form 1 January 2018; Accepted 16 January 2018

iterative wake model, or even a semi-empirical prescribed (non-iterative) rigid model may be applied. An example of the latter is the one proposed by Kerwin and Lee (1978), for use with a lifting surface description of the propeller blades. These wake models have been evolving and were recently applied to the design and analysis of propellers. Baltazar et al. (2012) presented a numerical wake alignment method implemented in combination with the panel method. Menéndez Arán and Kinnas (2014) proposed a propeller design optimization procedure that included a wake alignment scheme.

The development of the lifting line theory to horizontal axis turbines followed closely the one for propellers. As a matter of fact, the optimization criterion of Betz can be directly applied to turbine rotors and the corresponding potential flow solution, in line with the Goldstein solution, has been presented by Maekawa (1986). Further work on the optimization of horizontal axis turbines with a helical vortex model using Biot-Savart integration for the induced velocities has been carried out in Chattot (2003) and Sharpe (2003). The optimization problem using induction factors to avoid full Biot-Savart integration was presented by Falcão de Campos (2007). These authors considered rigid helicoidal wakes which limits the application of the lifting line theory to moderately loaded rotors. The introduction of a wake alignment scheme that models the axial variations of the wake pitch would certainly contribute to a more accurate computation of the induced velocities, axial force and power coefficients.

The purpose of this paper is to present a simplified vortex wake alignment scheme for the lifting line model, which can be easily applied in the analysis of turbine rotors. It follows the numerical approach of Baltazar et al. (2012). The wake geometry is defined by the axial and tangential velocities in multiple sections downstream of the rotor. The wake pitch is therefore allowed to vary along the streamwise direction. In this work the wake expansion has been neglected.

The lifting line theory coupled with the present wake alignment scheme is substantially simpler and computationally lighter than viscous flow models, as Reynolds-Averaged Navier-Stokes (RANS) solvers - see, for instance, Gharraee et al. (2016) and Stergiannis et al. (2016). Naturally, vortex models are limited to their range of applicability – the vorticity must be confined to small layers close to the surfaces. However, when this condition is met, it is possible to successfully characterize the kinematic field in the wake and calculate the velocity field near the blades. Under these conditions the theory is able to generate useful results for preliminary design studies.

In section 2 we present the mathematical formulation. The numerical method is described in section 3 where the wake alignment scheme and the iterative procedure are detailed. In section 4 the wake alignment algorithm was tested in the analysis of a wind turbine developed by the National Renewable Energy Laboratory (NREL) (Hand et al., 2001). The results of the wake alignment are presented for an increasing number of wake alignment sections located at different axial positions to test the model sensitivity to these parameters. In the computations the hub is modeled as an infinite cylinder (Kerwin, 2001) and forces are corrected for the section drag (Chattot, 2003; Falcão de Campos, 2007). The results are compared with experimental data (Sorensen et al., 2002). In section 5 final conclusions are drawn.

2. Mathematical formulation

Consider the rotor of a horizontal axis wind turbine with radius R and Z blades symmetrically placed around a cylindrical hub of radius r_h. The rotor is rotating with constant angular velocity $\vec{\omega}$ in a uniform inflow with velocity $\vec{U}$, aligned with the rotation axis. The fluid is assumed to be inviscid and incompressible. We introduce a Cartesian coordinate system (x, y, z) and a cylindrical coordinate system (x, r, θ) in a reference frame rotating with the turbine rotor. The flow is steady in the rotating reference frame and the relative velocity field is given by $\vec{U}_\infty = \vec{U} - \vec{\omega} \times \vec{r}$, as illustrated in Fig. 1.

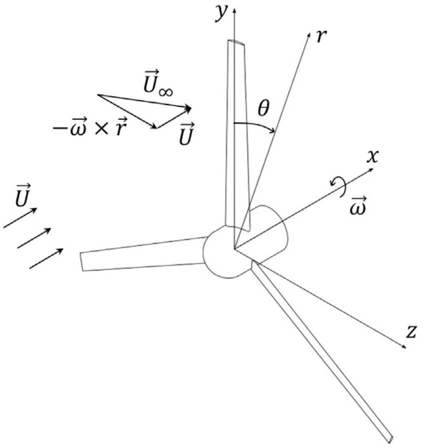

Fig. 1. Adopted coordinate system and inflow velocity field.

In the lifting line model each turbine blade is represented by a radial bound vortex, extending from the root to the blade tip and located at $\theta_k = 2\pi(k-1)/Z,\ k = 1, \ldots, Z$. The circulation along the lifting lines varies continuously, $\vec{\Gamma}(r) = -\Gamma(r)\vec{e}_r$, where $\vec{e}_r$ is the radial unit vector. Consequently, a semi-infinite vortex sheet is shed from each lifting line. Its intensity, $\vec{\gamma}$, results from Helmholtz theorems and is given by:

$$\vec{\gamma} = \frac{d\Gamma(r)}{dr}\vec{e}_s \tag{1}$$

where $\vec{e}_s$ is a unit vector tangent to the vortex sheet and aligned with the vortex filaments.

The vortex system corresponding to the first lifting line ($k = 1$) is illustrated in Fig. 2.

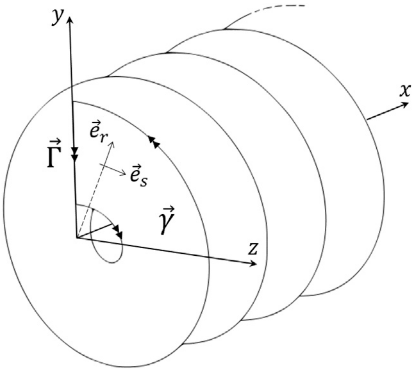

Fig. 2. Lifting line $k = 1$ and corresponding vortex sheet.

In order to establish a force-free vortex wake the vortex filaments must be aligned with the local velocity field:

$$\vec{\gamma} \times \vec{V} = 0 \tag{2}$$

where $\vec{V}$ is the total velocity.

This vortex system is the result of a non conservative force field acting on the fluid elements that are instantaneously coincident with the lifting lines. In fact, the only way to model lift in a non viscous fluid is by considering non conservative forces that introduce vorticity in the fluid flow. As expected, we may derive the resultant bound vortex and corresponding vortex sheet analytically, using Euler and mass conservation equations for incompressible flow - see, for example, Sparenberg (1995).