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Stochastic Fourier spectrum model and probabilistic information analysis for wind speed process

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ABSTRACT

From the view point of physical mechanism, stochastic dynamic excitations, such as earthquake and strong wind, subjecting on structures can be represented by a physical model with elemental random variables. In this article, a physical model for wind speed process is investigated to obtain the probability density function of wind speed process. Firstly, the stochastic Fourier spectrum, consisting of wave-number spectrum and phase spectrum, is introduced. The wave-number spectrum displays the energy distribution over frequency domain. The phase spectrum is viewed as the evolutionary result driven by characteristic velocity of air vortex. After that, the elemental variables are carefully selected in term of physical relation and the measurement data collected at a wind observation station is analyzed to gain the statistics of the elemental random variables. With the help of probability density evolution method (PDEM), the probability density information of wind speed process can be obtained. The comparison with the measurement data validates the effectiveness of the stochastic Fourier spectrum to simulate the wind speed process.

1. Introduction

For a large number of structures, such as high buildings and long span bridges, wind load is one of the most important dynamic excitations which may causes the loss of the serviceability or even totally failure of the structure. Because of highly stochastic and non-linearly properties, the fluctuating wind simulation becomes one of the most important issues in wind excitation modeling and lots of pioneer researchers have devoted their effort into this field.

In history, Von Karman was the earlier one to model the fluctuating wind. In his work, by introducing Gaussian process assumption, the concept of power spectrum density (PSD) was proposed (Von Karman, 1948). Then, the well-known Davenport spectrum was proposed in 1960s based on the records at different heights (Davenport, 1961). The following researchers, such as Kaimal, developed this approach to satisfy the requirements for different structures and terrain (Kaimal et al., 1972). Virtually, the information of PSD is not enough for description of the time-domain process since the initial phases are undefined. In general simulation schemes, such as spectral representation method (Shinozuka, 1971; Di Paola, 1998), it is often assumed that the phases are independent random variables uniformly distributed in the domain $[0, 2\pi]$. These works established the frame for simulating fluctuating wind on the base of power spectral density. However, in traditional methods, the probabilistic characteristics of fluctuating wind process are quite limited, where the information higher than 2nd order is omitted. Also, researches have revealed that for strong wind, such as typhoon, behavior of fluctuating wind process may distinctly depart from the Gaussian situation (Kareem, 1978; Gurley et al., 1996; Gong and Chen, 2014; Hui et al., 2017). These drawbacks arise from that the PSD models are not capable of describing the integrated probabilistic information of wind speeds. It is then desirable to obtain the probability density function which encompass the whole probabilistic information.

In fact, PSD model is a phenomenology based approach so that it is difficult to represent the behavior of fluctuating wind comprehensively. Therefore, physical way is needed for exquisite depiction. Along with the basic idea of physical stochastic system, it is some elemental variables in the physical model are uncontrollable that the process possesses the stochastic features (Li, 2006). Based on this idea, Li and Yan (2009) and Li et al. (2012) propose a stochastic Fourier function which stresses the key role of physical relation in the stochastic excitations modeling. According to the energy spectrum equation of turbulence, a bilinear amplitude spectrum is proposed by Li and Yan (2009) and Li et al. (2012) which display the distribution of energy spectra over inertial sub-range and energy containing sub-range. Then the phase spectrum is

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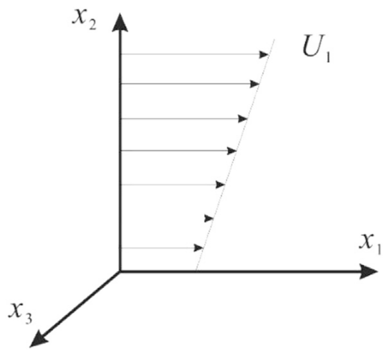

Fig. 1. Illustration of the homogeneous shear flow.

established with the help of the concept, starting-time of phase evolution, which gives a physical description of the phase evolution in wind speed process (Li et al., 2013). The analysis using the measured data collected in east China validated approach preliminarily (Yan, 2011). On the other hand, the newly developed probability density evolution method provides a useful tool to capture the probability density function of stochastic dynamic system (Li and Chen, 2008, 2009). Thus, This article is dedicated to propose a new approach, based on stochastic Fourier spectrum and probability density evolution method, to obtain the probability density function and consequently other probabilistic information of wind speed process, which can be of significance in reliability assessment of engineering structures. Furtherly, the model is validated by the data collected from an strong wind observation station.

The contents are arranged as following: the stochastic Fourier spectrum mainly deduced from the dynamic equation for the energy spectrum is introduced and updated in section 2; the statistics of the elemental random variables are modeled using the wind measurements collected in Xiamen in section 3; section 4 introduces the probability density evolution method which will be applied to research the probability density evolution of wind speed process; section 5 provides the probability density distribution of the wind history derived from the physical model integrated with the probability density evolution method, and the contrast of the simulation and observations demonstrates the effectiveness of this model; moreover, some important statistical characteristics deduced from stochastic Fourier spectrum are also inspected in section5.

2. Physical model for fluctuating wind speeds

2.1. Stochastic Fourier spectrum

From the view point of physical stochastic system, a physical model with elemental variables can be established to represent the process logically. For instance, let $u(\boldsymbol{\Theta}, t)$designate a record of fluctuating wind speeds while $\boldsymbol{\Theta}$ is the elemental variables which determine the process through the definitive physical model $u(\boldsymbol{\Theta}, t)$. Virtually, a Fourier transform of $u(\boldsymbol{\Theta}, t)$, denoted by $F(\boldsymbol{\Theta}, n)$, contains the same information as the original process $u(\boldsymbol{\Theta}, t)$ according to Winner-Khintchine formula:

$$F(\boldsymbol{\Theta}, n) = \frac{1}{\sqrt{T}} \int_{-\infty}^{\infty} u(\boldsymbol{\Theta}, t) \mathrm{e}^{-\mathrm{i}2\pi nt} \mathrm{d}t \tag{1}$$

$$u(\boldsymbol{\Theta}, t) = \sqrt{T} \int_{-\infty}^{\infty} F(\boldsymbol{\Theta}, n) \mathrm{e}^{-\mathrm{i}2\pi nt} \mathrm{d}n \tag{2}$$

On account of this equivalence, the Fourier function $F(\boldsymbol{\Theta}, n)$ can be defined as the physical model of fluctuating wind speeds. It is believed the source of the randomness of fluctuating wind lies in the randomness of $\boldsymbol{\Theta}$. Once the Fourier function is established and the distribution of elemental variables is identified, the model can be used to carry out the analysis for the stochastic system.

Generally, the Fourier function $F(\boldsymbol{\Theta}, n)$ is a complex and can be rewritten as the product of two parts:

$$F(\boldsymbol{\Theta}, n) = |F(\boldsymbol{\Theta}, n)| \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{\Theta}, n)} \tag{3}$$

where $|F(\boldsymbol{\Theta}, n)|$ is the amplitude spectrum and $\varphi(\boldsymbol{\Theta}, n)$ the phase spectrum.

It is commonly recognized that the amplitude describes the distribution of energy over frequency domain where the phase spectrum controls the shape of the time history. The amplitude spectrum can be related to wave-number spectrum if Taylor's hypothesis is accepted:

$$\begin{aligned} |F(\boldsymbol{\Theta}, k)| &= \sqrt{\frac{U}{2\pi}} |F(\boldsymbol{\Theta}, n)| \\ k &= \frac{2\pi}{U} n \end{aligned} \tag{4}$$

To model the wave-number spectrum rationally, basic knowledge of energy spectrum of turbulence is needed. The energy spectrum equation of turbulence (See Fig. 1) reads (Hinze, 1975):

$$\frac{\partial}{\partial t} E(k) + \zeta(k) \frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} = F_3(k) - 2\nu k^2 E(k) \tag{5}$$

where $E(k) = |F(k)|^2$ is the energy spectrum of turbulence in wave-number domain, $F_3(k)$denotes the Fourier transform of the third order velocity correlation which transport the energy from low wave-number domain to high wave-number domain, $\zeta(k)$ represents the energy transportation caused by the shear in main flow.

Integrate the equation (5) from 0 to k over wave-number domain, we can obtain:

$$\varepsilon = 2\nu \int_0^k k^2 E(k) \mathrm{d}k - \frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} \int_k^{\infty} \zeta(k) \mathrm{d}k - \int_0^k F_3(k) \mathrm{d}k \tag{6}$$

where $\int_0^{\infty} \zeta(k) \mathrm{d}k = \overline{u_1 u_2}$ and $\varepsilon = -\overline{u_1 u_2} \frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2}$are utilized.

To solve the integral equation (6), the Heisenberg's theorem of energy transfer is introduced (Katul and Chu, 1998):

$$\int_0^k F_3(k) \mathrm{d}k = -2\alpha' \int_k^{\infty} \sqrt{\frac{E(k)}{k^3}} \mathrm{d}k \int_0^k k^2 E(k) \mathrm{d}k \tag{7}$$

where α' is a constant.

In the inertial subrange, Tchen's assumption is adopted since the interaction between the turbulence and the main flow is relatively weak (Katul and Chu, 1998):

$$-\frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} \int_k^{\infty} \zeta(k) \mathrm{d}k = \alpha'' \left(\frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} \right)^2 \int_k^{\infty} \sqrt{\frac{E(k)}{k^3}} \mathrm{d}k$$

where α'' is another constant. Meanwhile, a negligible shear rate can be assumed. As a result, an approximate solution of the integral equation (6) in the inertial subrange can be derived as follows:

$$E(k) = \left(\frac{8}{9\alpha'} \right)^{2/3} \varepsilon^{2/3} k^{-5/3}$$

In the energy containing subrange, an intense interaction between the turbulence and main flow exists and dominates the integral equation (6) comparing to other terms, thus we have:

$$-\frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} \int_k^{\infty} \zeta(k) \mathrm{d}k = \alpha'' \frac{\mathrm{d}\overline{U_1}}{\mathrm{d}x_2} \left[2 \int_0^k k^2 E(k) \mathrm{d}k \right]^{1/2} \int_k^{\infty} \sqrt{\frac{E(k)}{k^3}} \mathrm{d}k$$

which combined with (7) leads to:

$$E(k) = \frac{1}{\alpha''} \frac{\varepsilon}{\frac{dU_1}{dx_2}} k^{-1}$$

To conclude, the solution of equation (6) over energy containing subrange and inertia sub-range have the following form: