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Modelling of distributed aerodynamic pressures on bridge decks based on proper orthogonal decomposition

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ABSTRACT

Stress-level buffeting analysis is necessary for evaluating stress-related performance and safety of long-span bridges. A stress-level buffeting analysis requires knowledge of characteristics of distributed aerodynamic pressures on a bridge deck. This paper, therefore, presents a framework for modelling distributed aerodynamic pressures on the surfaces of a bridge deck in terms of the proper orthogonal decomposition (POD) method. The characteristics of aerodynamic pressure modes, such as covariance modes, principal coordinates, pressure modal coefficients, and pressure modal admittance functions are introduced in the formulation. Wind tunnel pressure tests of a sectional motionless deck model were conducted to identify the characteristics of distributed aerodynamic pressures on a twin-box bridge deck as a case study. The pressure modal admittance functions were identified by using a colligated least-square method. The results show that the higher-order POD pressure modes provide a very limited contribution to the aerodynamic pressures and can be truncated without notable loss of accuracy. The aerodynamic pressures distributed over the bridge deck surface can be well represented by a superposition of a limited number of POD pressure modes that include the first two pressure modes. The use of pressure modal admittance functions greatly simplifies the identification of many individual aerodynamic pressure admittance functions.

1. Introduction

With the increasing span length of modern cable-supported bridges, buffeting-induced fatigue damage to steel decks has attracted more and more attention in the field of wind engineering (Gu et al., 1999; Xu et al., 2009). The estimation of fatigue damage to key components of a bridge deck requires the knowledge of buffeting-induced stress response and distribution. Stress-level buffeting analysis is therefore crucial in fatigue evaluation of long-span bridges (Liu et al., 2009).

Traditional buffeting analysis methods (e.g. Scanlan, 1978; Chen et al., 2000; Xu et al., 2000; Larsen and Larose, 2015) are almost all force-based and performed using simple finite element (FE) models that use beam elements of equivalent sectional properties to model a bridge deck. These methods and the associated beam element models are not sufficient for accurate stress analyses. Advanced FE models built in detailed geometry with plate/shell/solid elements, which often involve multi-scale modelling (e.g. Chan et al., 2009; Duan et al., 2011; Kong et al., 2012; Zhu et al., 2015), have been recently developed to obtain accurate stress responses. Nevertheless, in order to carry out an accurate analysis of buffeting-induced stress response of long-span bridges with advanced FE models, it is imperative to take into account the cross-sectional distribution of aerodynamic pressures rather than the cross-sectional aerodynamic forces. Many studies have been conducted in the past two decades to acquire pressure data from pressure tests of motionless sectional deck models (e.g. Hui, 2006; Jakobsen, 1997; Larose, 1997; Yan et al., 2016). Most of these studies focused on the admittance functions of aerodynamic forces. Zhu and Xu (2014) and Zhu et al. (2016a,b) have recently investigated the characteristics of distributed aerodynamic pressures on the surface of a motionless twin-box bridge deck. The admittance functions of distributed aerodynamic pressures have been identified. However, the admittance functions have been introduced to every single pressure point on a deck section in their work, which demands an impractically immense amount of work for the determination of aerodynamic pressures. Therefore, it is desirable to have a new method

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that can describe the sectional distribution of the unsteady aerodynamic pressures with only a few to-be-identified admittance functions.

To further investigate the distribution of wind-induced aerodynamic pressures around a bridge deck and to fully capture the distributing pattern and characteristics of aerodynamic pressures yet with only a small number of to-be-identified parameters, the proper orthogonal decomposition (POD) method is employed as a powerful mathematical tool in this study. The POD method has been applied to random fields since the 1970s (Lumley, 1970) and has been introduced into wind engineering later on as an effective method to understand the inner mechanism of the fluid-structure interaction. While most of the related researches focused on the surface pressure field of buildings (e.g. Chen and Kareem, 2005; Katsumura et al., 2007; Tamura et al., 1999), the studies on the fluctuating pressure field over the cross-section of vibrating square cylinder models have been carried out (de Grenet and Ricciardelli, 2004; Hoa and Tamura, 2008). The POD method has also been applied to analyse random pressure fields around a vibrating bridge box deck (de Grenet and Ricciardelli, 2004), with the aim of decoupling the random pressure fields into a number of uncorrelated random scalar processes.

This study presents a framework to model distributed unsteady aerodynamic pressures over a bridge deck section with a few pressure modes in terms of the POD method. The pressure modal admittance functions are introduced to consider the unsteadiness of aerodynamic pressures. As traditional equivalent admittance method (EAM) (Larose, 1999; Gu and Qin, 2004) cannot separate *u*- and *w*-related admittance functions of aerodynamic forces, a colligated least-square method, proposed by Zhu et al. (2016a,b), are applied to the auto and cross spectra of the measured wind speeds and pressures to identify and distinguish the two components of pressure modal admittance functions. Wind tunnel pressure tests on a sectional motionless twin-box bridge deck model were carried out as a case study to validate the proposed method. The characteristics of distributed aerodynamic pressures, such as pressure coefficients, covariance modes, principal coordinates, pressure modal coefficients, and pressure modal admittance functions are discussed.

2. Modelling of distributed aerodynamic pressures

This section presents a framework to describe the distributed unsteady aerodynamic pressures over a bridge deck section based on the POD method. Aerodynamic pressure covariance modes, pressure principal coordinates, pressure modal coefficients, and pressure modal admittances are introduced in the modelling.

2.1. Pressure modes and modal admittances

Based on the linear quasi-steady assumption, the aerodynamic pressures on the surface of a bridge deck section can be described by the following equation (Zhu and Xu, 2014).

$$P_{i,b}(t) = \frac{1}{2}\rho\overline{U}\left[2C_{p_i}(\theta)u(t) + C'_{p_i}(\theta)w(t)\right] \tag{1}$$

where ρ is the density of air; $\overline{U}$ is the mean speed of incoming wind; $u(t)$ and $w(t)$ are the longitudinal and vertical turbulent components, respectively; $C_{p_i}(\theta)$ and $C'_{p_i}(\theta)$ are the pressure coefficient and its derivative defined in the structural coordinate system; and θ is the mean attack angle of incident wind.

Furthermore, admittance functions are introduced into the expressions of distributed aerodynamic pressures to consider unsteady aerodynamic effects. Since the admittance function is frequency-based parameter, the aerodynamic pressures can be presented in mixed time-frequency domain as follows

$$P_{i,b}(t) = \frac{1}{2}\rho\overline{U}\left[2C_{p_i}(\theta)\chi_{p_iu}(\omega)u(t) + C'_{p_i}(\theta)\chi_{p_iw}(\omega)w(t)\right] \tag{2}$$

where $\chi_{p_iu}(\omega)$ and $\chi_{p_iw}(\omega)$ are the pressure admittance functions of the aerodynamic pressure at the *ith* surface point of the bridge deck section with respect to the fluctuating wind speeds *u* and *w*, respectively.

The use of Eq. (2) to describe distributed aerodynamic pressures requires the identification of a large number of admittance functions of aerodynamic pressures, which is time-consuming and not reliable. To circumvent this difficulty, the POD method is employed in this study. With the POD, aerodynamic pressures over a bridge deck section can be represented as the superposition of the products of a series of principal coordinates and uncorrelated covariance modes, and the distributed aerodynamic pressures can be expressed accordingly as

$$\boldsymbol{P}_b(t) = \boldsymbol{\Phi}\mathbf{A}_b(t) = \sum_{j=1}^{N}\alpha_{j,b}(t)\boldsymbol{\phi}_j \tag{3}$$

where $\boldsymbol{P}_b(t) = \{P_{1,b}(t), P_{2,b}(t), \cdots, P_{i,b}(t)\cdots, P_{N,b}(t)\}^T$ is the vector of distributed aerodynamic pressures; $\mathbf{A}_b(t) = \{\alpha_{1,b}(t), \alpha_{2,b}(t), \cdots, \alpha_{j,b}(t)\cdots, \alpha_{N,b}(t)\}^T$ is the principal coordinate vector of aerodynamic pressure modes; $\boldsymbol{\Phi} = \{\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_j\cdots, \boldsymbol{\phi}_N\}$ represents the corresponding covariance mode matrix; and N is the total number of pressure points in the measured bridge deck section.

The covariance modes $\boldsymbol{\Phi}$ can be found by solving the eigenvalue problem of the covariance matrix (Bienkiewicz et al., 1993; Jeong et al., 2000).

$$\boldsymbol{R}^*_{p,b}\boldsymbol{\Phi}^* = \boldsymbol{\Phi}^*\boldsymbol{\Lambda} \tag{4}$$

where $\boldsymbol{R}^*_{p,b} = \boldsymbol{W}^{1/2}\boldsymbol{P}_b\boldsymbol{P}_b^T\boldsymbol{W}^{1/2}$ represents the modified covariance matrix of distributed aerodynamic pressures; $\boldsymbol{W} = diag\{\delta_1, \delta_2, \cdots, \delta_i\cdots, \delta_N\}/\overline{\delta}$ is a dimensionless weighting vector considering the characteristic length of each pressure while $\overline{\delta}$ is the mean characteristic length; $\boldsymbol{\Lambda} = diag\{\lambda_1, \lambda_2, \cdots, \lambda_j\cdots, \lambda_N\}$ and $\boldsymbol{\Phi}^*$ are the corresponding eigenvalue matrix and the eigenvector matrix, respectively. The sought eigenvectors can be obtained by

$$\boldsymbol{\Phi} = \boldsymbol{W}^{1/2}\boldsymbol{\Phi}^* \tag{5}$$

The *jth* principal coordinate $\alpha_{j,b}(t)$ can then be calculated with the measured aerodynamic pressures and expressed by the following equation if admittance functions of aerodynamic pressures are not considered:

$$\alpha_{j,b}(t) = \sum_{i=1}^{N}P_{i,b}(t)\phi_{ij} = \frac{1}{2}\rho\overline{U}\sum_{i=1}^{N}\left[2C_{p_i}(\theta)u(t) + C'_{p_i}(\theta)w(t)\right]\phi_{ij} \tag{6}$$

Eq. (6) can be simplified by introducing the pressure modal coefficient $C_{\alpha_j}(\theta)$ and its derivative $C'_{\alpha_j}(\theta)$ as

$$\alpha_{j,b}(t) = \frac{1}{2}\rho\overline{U}\left[2C_{\alpha_j}(\theta)u(t) + C'_{\alpha_j}(\theta)w(t)\right] \tag{7}$$

$$C_{\alpha_j}(\theta) = \sum_{i=1}^{N}C_{p_i}(\theta)\phi_{ij} \tag{8}$$

$$C'_{\alpha_j}(\theta) = \sum_{i=1}^{N}C'_{p_i}(\theta)\phi_{ij} \tag{9}$$

To take into account the unsteady aerodynamic effect, admittance functions are now introduced into the principal coordinate of each aerodynamic pressure mode in a similar way as those for aerodynamic forces or pressures, and they are termed as “aerodynamic pressure modal admittance functions”. In this way, the number of admittance functions can be reduced significantly, compared with admittance functions of aerodynamic pressures. The aerodynamic pressure modal principal coordinate $\alpha_{j,b}(t)$ is then expressed as