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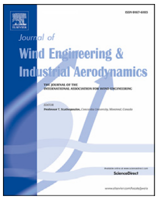

Analytical and empirical models of tornado vortices: A comparative study

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ABSTRACT

Tornadoes are generally defined as rotating columns of air characterized by small size, axisymmetry and short duration. Thus, the nature of tornadic wind load is basically different from that of synoptic wind. As direct measurements of velocities are very difficult and limited, lots of empirical and theoretical expressions of tornadic wind have been proposed. In the present study, characteristics of several numerical expressions were investigated, and it was found that the characteristics of velocity fields vary widely from model to model, making it difficult to choose one that is generally applicable.

1. Introduction

During the last ten decades, there has been great progress in the establishment of design wind loads and wind-resistant designs of buildings and structures for synoptic wind mainly caused by monsoon and/or large-scale storms. However, there are strict safety requirements for the protection of nuclear power plants and electric transmission towers due to the serious consequences of an accident. Thus, the need to establish design wind loads and wind-resistant designs for non-synoptic wind, especially for tornadoes, has attracted a lot of interest for engineers and researchers in the wind engineering field. A tornado is generally defined as a rotating column of air that is pendant from a cumuliform cloud in contact with the earth's surface, and is often visible as a funnel cloud and/or circulating debris/dust near ground level (http://glossary.ametsoc.org/wiki/Tornado). Tornadoes can occur anywhere around the world, including Japan, Bangladesh, Australia, Britain, among others. In the U.S., Tornado Alley was a synonym for the most tornado-prone region. However, these days it is known that there is a lot of tornado activity in Southeast region, called Dixie Alley, as reported in Dixon et al. (2011). In Japan, 40–50 tornadoes occur annually, most of them near the coastline (Tamura et al., 2015). Tornadoes are known to be the most violent and damaging meteorological phenomena on the earth's surface to human life and property. Their vortices usually rotate cyclonically and some of them contain secondary vortices. A severe tornado brings extremely strong winds accompanied by sudden pressure change causing death and serious damage to or collapse of structures along its path.

As tornadoes are characterized by small size, axisymmetry, high velocity, high vorticity and short duration, the nature of tornadic wind load is basically different from that of large-scale storms. The main ways in which tornadoes are different (Wen and Ang, 1975) are i) the probability of a certain place being damaged is small: the mean recurrence interval for a tornado striking a point is 1000years (Thom, 1963), ii) the durations of wind action is short because of their rapid change in both magnitude and direction, and iii) effects of updraft and pressure change could become important. Thus, the conventional analysis and wind-resistant design is clearly not applicable to tornadic winds. Since tornadoes move fast and their courses are unpredictable, the study of tornadoes by measuring wind velocities and pressure changes has been always difficult and limited. Furthermore, a debris wall within tornadoes makes field observations difficult (Baker and Sterling, 2017). Although tornadoes have very different characteristics during their lifetimes, the following are generally recognized by field observations (Baker and Sterling, 2017). i) Tornadoes range from a very simple single cell with one radial inflow and vertical updraft to multiple cell structures with updrafts and downdrafts at different radii, some extending down to the ground and some not. ii) Tornadoes are driven by atmospheric temperature and density differences, thus creating buoyant forces. iii) The nature of tornadoes varies with time, so they have different characteristics during their lifetimes. iv) There is a boundary layer region near the ground that is dominated by local ground roughness. Recently, various methods such as laboratory experiments, analytical modeling, computational simulations, mobile radar and in-situ measurements have been implemented to gain insight into the problem and to provide comprehensive understanding.

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Nomenclature

A_m and B_m	constant used in Fujita model
F_r	body force in r direction
F_z	body force in z direction
F_θ	body force in θ direction
H_i	inflow layer used in Fujita model [m]
K	ratio of two random radial distances (r_1/r_2)
K_B	constant used in Baker model
K_{BR1} and K_{BR2}	constant used in Burgers-Rott model
K_{S1} and K_{S2}	constant used in Sullivan model
P	pressure at radial distance r [Pa]
P_i	pressure within the boundary layer in Kuo-Wen model [Pa]
P_o	pressure above the boundary layer in Kuo-Wen model [Pa]
P_∞	pressure at infinity [Pa]
P_0	pressure at the tornado center [Pa]
S	swirl ratio in Baker model
U	radial velocity [m/s]
U_i	radial velocity within the boundary layer in Kuo-Wen model [m/s]
$U_{\max}$	maximum radial velocity in Baker model [m/s]
U_o	radial velocity above the boundary layer in Kuo-Wen model [m/s]
V	tangential velocity [m/s]
V_i	tangential velocity within the boundary layer in Kuo-Wen model [m/s]
$V_{\max}$	maximum tangential velocity [m/s]
V_o	tangential velocity above the boundary layer in Kuo-Wen model [m/s]
$V_{o,\max}$	maximum tangential velocity above the boundary layer in Kuo-Wen model [m/s]
V_1	tangential velocity at r_1 [m/s]
V_2	tangential velocity at r_2 [m/s]
W	vertical velocity [m/s]
W_i	vertical velocity within the boundary layer in Kuo-Wen model [m/s]
W_o	vertical velocity above the boundary layer in Kuo-Wen model [m/s]
a	velocity gradient in Burgers-Rott and Sullivan model [1/s]
b	function representing the velocity fluctuations outside the core in Kuo-Wen model
k and k_0	constant used in Fujita model
n	ratio of inner and outer core radii in Fujita model (core ratio, r_n/r_o)
r	radial coordinate, radial distance from the tornado center [m]
r_c	radius of maximum tangential velocity [m]
r_{co}	the radius of the maximum tangential velocity above the boundary layer in Kuo-Wen model [m]
r_m	radius at maximum radial velocity in Baker model [m]
r_n	inner core radius in Fujita model [m]
r_o	outer core radius in Fujita model [m]
r_1	random radial distance [m]
r_2	random radial distance [m]
t	time [s]
z	vertical coordinate, vertical height [m]
z_m	height at maximum radial velocity in Baker model [m]
Γ_∞	vortex circulation at infinity [m^2/s]
α	crossing angle between the direction of flow and tornado circle at crossing point in Fujita model
α_o	crossing angle α at $r = r_o$
β	power-law index governing the sharpness of the tangential velocity in Vatistas model
θ	tangential coordinate
ε	decay index expressing the degree of decrease in the outer region in modified Rankine model
ζ	vertical vorticity [1/s]
η	decay parameter in Wood-White model
η_{KW}	ratio of vertical height and boundary layer thickness in Kuo-Wen model (z/δ)
δ	boundary layer thickness in Kuo-Wen model [m]
δ_∞	boundary layer thickness at infinity in Kuo-Wen model [m]
κ	growth parameter in Wood-White model
λ	size parameter in Wood-White model
ν	kinematic viscosity [m^2/s]
ν_e	eddy viscosity [m^2/s]
ρ	air density [kg/m^3]

Tangential velocities in tornadoes are often approximated by continuous functions that are zero at the tornado center, increase to a maximum at some radius, and then decrease asymptotically to zero infinitely far from the center. A lot of theoretical and empirical expressions for tangential velocity have been proposed by many researchers with various backgrounds. Among them, the idealized and inviscid Rankine model has been widely implemented as a first approximation. But several shortcomings of the Rankine model have led to better approximations. The parameters in numerical expressions should contain useful information about the physical structure of tornadoes that can be used to determine tornado structures when there are incomplete data. The importance of the choice of tornadic model can be understood from the existing studies. Kobayashi et al. (2015) reported that a modified Rankine model gives the best fit of tangential velocity to the field measurement data, while the Burgers model gives the largest tip displacement of transmission tower when compared with Rankine and Fujita model (Ishizaki et al., 2016). Even though the modified Rankine model gives good agreement with field measurement data, structural engineers may choose the Burgers model for safe design. As exactly the same numerical models were not used in those two studies, it is difficult to make a general statement, but discussing and addressing the importance of choosing an appropriate numerical model is of great interest.

A lot of empirical and/or theoretical numerical expressions have been derived for tornadic wind, but no detailed comparative studies have been conducted so far. The objective of this paper is to investigate the characteristics of several numerical expressions, either empirical or theoretical and one-dimensional or three-dimensional, which are frequently employed in the wind-resistant design of structures against tornadoes.

2. Equations of motion in cylindrical coordinates

When the radial, tangential, and vertical velocity components are denoted by U, V and W, respectively, as shown in Fig. 1, the momentum (Navier-Stokes) equations and mass conservation (continuity) equation for an incompressible fluid with constant eddy viscosity (υ_e) in cylindrical coordinates (r, θ, z) are

Radial (r-direction) momentum equation:

$$\begin{aligned}&\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial r}+\frac{V}{r}\frac{\partial U}{\partial \theta}+W\frac{\partial U}{\partial z}-\frac{V^2}{r}\\&=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu_e\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}+\frac{\partial^2 U}{\partial z^2}-\frac{U}{r^2}-\frac{2}{r^2}\frac{\partial V}{\partial \theta}\right\}+\frac{1}{\rho}F_r\end{aligned} \tag{1}$$