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Short Note

Short communication: On the Gaussian-Exponential Mixture Model for pressure coefficients

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ABSTRACT

It is argued that the probability density function for surface pressures on buildings is expected to be skewed Gaussian with exponential tails. A Skewed Gaussian-Exponential Mixture Model for wind pressures on buildings is demonstrated using data from various published sources. This model is simple to apply, gives a good fit to both body and tails of the density function and generates the required exponential tails with a constant relative error.

1. Introduction

This paper reports unfunded, curiosity-driven research using data mined from recently published papers to find a model for pressure coefficients on buildings that is physically realistic and accurate in both body and tails.

Peterka and Cermak (1975) observed that the probability density function (PDF) of the time history of local pressures on building models fell into two distinct categories. In regions of attached flow, principally the windward face, the PDFs were close to Gaussian, but negatively skewed. In regions of separated flow the PDFs also became exponential in the tails, especially in the suction tails. In discussing their paper, Davenport (1976) emphasised the importance of accounting for these exponential tails in design. Experimental data illustrating these two categories are shown in Figs. 1 and 2 by the open circle symbols, reproduced from Mayne and Cook (1978). Holmes (1981) showed that the skewness of the PDF on the windward face could be accounted for by including the squared terms on the turbulent velocities in the Bernoulli equation. Holzer and Siggia (1993) laid out the analytical arguments to show that static pressure fluctuations in a Gaussian velocity field with a Kolmogorov $-5/3$ decay spectrum will always be skewed and have exponential tails.

To account for the skew, Sadek and Simiu (2002) used a three-parameter Gamma distribution to fit mildly and to strongly non-Gaussian data. This distribution is continuously transitional between the Weibull and Gamma distributions. However, only the positive-going tail is (slowly) asymptotic to exponential, unless the distribution is reversed when this applies only to the negative tail. Quan et al. (2014) reviewed development of this field from Peterka and Cermak, through Holmes, to current translation methods using truncated Edgeworth series and Hermite polynomials. They conclude that PDFs of wind pressures are difficult to fit to a single function because there is no consistency in the wind pressure behaviour at different locations and that the Gamma distribution cannot satisfactorily fit both tails. The translation method of Kwon and Kareem (2011) and later methods, reviewed by Peng et al. (2014) and Okada et al. (2014) using Edgeworth series, based on Hermite polynomials derived from only the first four moments, are deficient in the far tails because these are essentially only the first four terms of a Taylor series expansion around the mean. In general, Edgeworth series are not guaranteed to generate proper PDFs as they may not integrate to 1 and can contain negative values in the far tails, but implementation using Hermite polynomials avoids these deficiencies (Yang et al., 2013). Okada et al. (2014) also apply a Gaussian mixture model (GMM) to introduce skew, which resolves some of these deficiencies, but the accuracy of the tails is again limited by the number of Gaussians used in the mixture.

2. Addressing the physics

The methods cited above are general-purpose methods applied to a specific physical problem. The Taylor-series based methods give an absolute error that reduces as the number of terms increases. This

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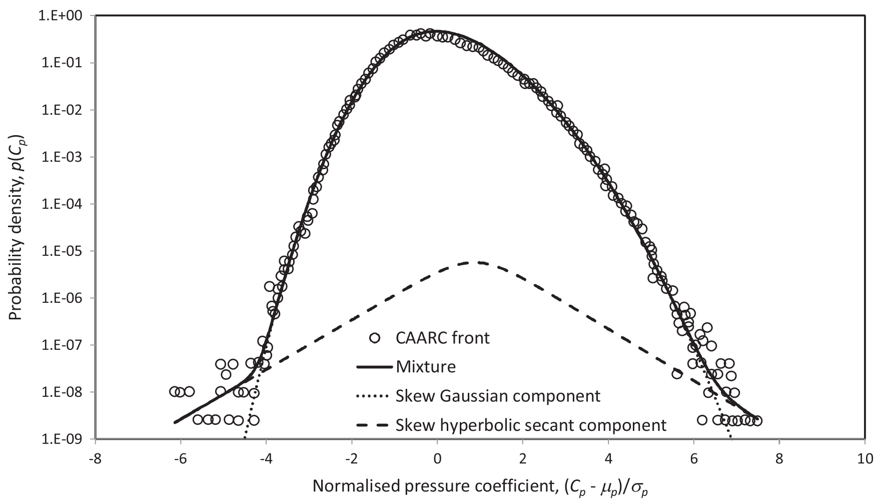

Fig. 1. PDF of pressure coefficient on front face of CAARC model (from Mayne and Cook, 1978).

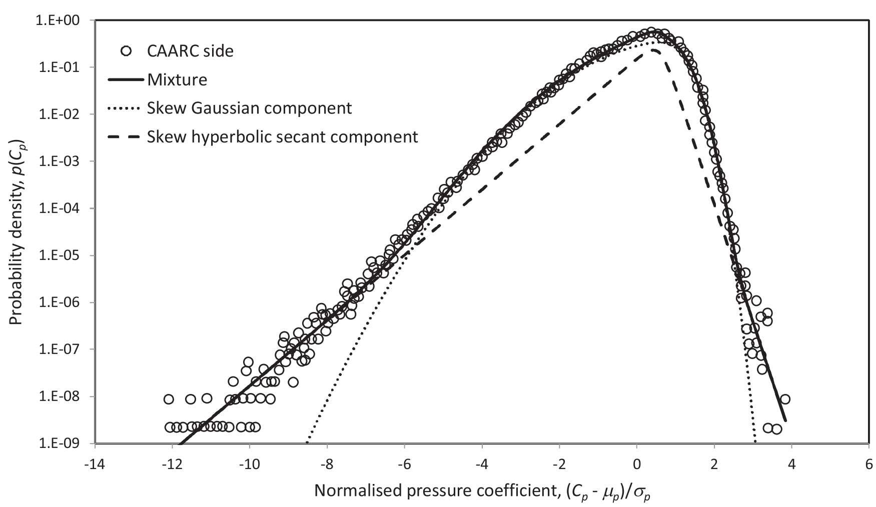

Fig. 2. PDF of pressure coefficient on side face of CAARC model (from Mayne and Cook, 1978).

may be satisfactory in the body of the PDFs, but gives an increasing relative error in the tails. A better approach is to tailor the method to the specific problem by representing the tails directly by an exponential, resulting in a constant relative error. Consider again the two categories of Peterka and Cermak (1975) corresponding to attached and separated flow zones around the building. As the flow is well subsonic, every velocity fluctuation affects the whole flow to some degree. While either category may be dominant, the other category will always have some influence, so that a mixture model is indicated by the physics.

In a mixture, the PDF $p(x)$ is modelled by:

$$p(x) = \sum_{i=1}^{n} f_i p_i(x) \tag{1}$$

where $p_i(x)$ is the PDF of a physical component and $0 \le f_i \le 1$ is the relative frequency of the i-th out of n components, such that $\sum_{i=1}^{n} f_i = 1$. In a mixture, the mean μ, standard deviation σ, skewness γ, and kurtosis κ, obtained from the overall moments is not representative of any one component. The aim here is to represent each physical component by a single PDF, $p_i(x)$, treating the exponential tails as a separate component. In the simplest case one component represents the body and another represents the tails, but more complex cases of multi-modal distributions caused by switching or reattaching flows may require more components.

3. The Skew Gaussian-Exponential Mixture Model

The Skew Gaussian-Exponential Mixture Model (SGEMM) explicitly addresses the issues of the Gaussian body, the exponential tails, and the skewness inherent in the PDFs of pressure by using a mixture of two forms of skewed distribution:

1. The skew-Gaussian distribution proposed by Azzalini (1985) for the body of the PDF.
2. The skew-Hyperbolic Secant distribution proposed by Cook (2014) for the tails.

3.1. *The skew-Gaussian distribution*

The skew-Gaussian (SG) distribution is given by

$$p(x) = 2\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\frac{x-\xi}{\omega}\right) \tag{2}$$

where ϕ and Φ are the standard Gaussian PDF and CDF, respectively, ξ is a location parameter, ω is a scale parameter and α is a skewness parameter. The form of Eq. (2) is shown in Fig. 3(a) on linear axes and (c) on semi-logarithmic axes for various values of α. The direction of skew depends on the sign of α. When $\alpha=0$ Eq. (2) reverts to the standard Gaussian PDF, giving the familiar bell-shaped curve in (a) and a parabola in (c).