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Development and validation of a new two-dimensional wake model for wind turbine wakes

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ABSTRACT

A new two-dimensional (2D) wake model is developed and validated in this article to predict the velocity and turbulence distribution in the wake of a wind turbine. Based on the classical Jensen wake model, this model is further employing a cosine shape function to redistribute the spread of the wake deficit in the crosswind direction. Moreover, a variable wake decay rate is proposed to take into account both the ambient turbulence and the rotor generated turbulence, different from a constant wake decay rate used in the Jensen model. The obtained results are compared to field measurements, wind tunnel experiments, and results of an advanced $k-\omega$ turbulence model as well as large eddy simulations. From the comparisons, it is found that the proposed new wake model gives a good prediction in terms of both shape and velocity amplitude of the wake deficit, especially in the far wake which is the region of interest for wind farm development projects.

1. Introduction

The Jensen wake model (Jensen, 1983; Katic et al., 1986), also named as the PARK wake model, is one of the most widely used analytical wake models, which has been implemented in commercial codes such as WAsP, GH WindFarmer, WindPRO and OpenWind. Based on the law of global momentum conservation, the Jensen model provides a mathematical expression of the wake velocity deficit to describe the wake of a single wind turbine. In this model, the wake width is assumed to linearly expand downstream with a wake decay constant (k), which is an empirically determined parameter. The suggested value of k in the literature (Barthelmie et al., 2006) is 0.075 for onshore cases and 0.05 for offshore applications. The Jensen model is easy to code with only a few input parameters. Despite its simplicity, the Jensen model has been proved to provide an acceptable representation of the wake behavior (Crespo et al., 1999; Porte Agel et al., 2013; Schlez et al., 2003). For single wake prediction, the results predicted by the Jensen model (Magnusson and Smedman, 1999) are in reasonable agreement with available data. Barthelmie et al. (2009) evaluated the Jensen model and five other engineering models against a set of experiments from an offshore wind farm. In addition, a comparison of different wake models (Barthelmie et al., 2006) shows that there is no particular difference between the analytical models and sophisticated models in term of accuracy. All these tests and validations have proved that even the Jensen model is old and simple, it can still exhibit a good, yet not perfect, match with the measured data.

The Jensen wake model is a one-dimensional (1D) model because the wake speed is assumed to be the only variable in function of the downwind distance, meaning that the predicted wake velocity profile along the crosswind direction at a certain downstream position is assumed to be uniform. On the other hand, the theoretical knowledge and field measurements (Vermeer et al., 2003) have pointed out that the assumption of the top-hat distribution in the 1D Jensen model is not realistic. In addition, it is also claimed by Katic et al. (1986) that the Jensen model gives an estimation of the output energy rather than describing the velocity field accurately due to the top-hat assumption.

In this article, a two-dimensional (2D) wake model (named as 2D Jensen model in this article) is presented by distributing the velocity profile in the cross section with a cosine shape instead of the top-hat shape in the standard Jensen model. Moreover, by taking into account the effect of turbulence on the wake recovery, the wake decay parameter k becomes a variable depending on both the atmospheric and the rotor generated turbulence, and also the downstream distance from the wind turbine. For this

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purpose, an analytical model for estimating the added turbulence generated by the presence of the wind turbine is proposed at first and then compared with some other engineering models to assess its validity.

The structure of this article is organized as follows: In Section 2, a brief review of the Jensen model is presented. After that, the 2D Jensen model is derived in Section 3. The developed model is then applied in several test cases to validate its accuracy and generality, including different types of wind turbines under different operating conditions and various atmospheric turbulence levels. Results are presented and compared with wind tunnel measurements, field measurements as well as numerical results from an advanced CFD (Computational Fluid Dynamics) method in Section 4. Finally conclusions and perspectives are given in Section 5.

2. Standard Jensen wake model

To derive the 2D Jensen model, a brief introduction of the 1D Jensen model is necessary. The Jensen model is based on the assumption that the wake velocity at a given downwind position can be expressed in terms of turbine's thrust coefficient C_T and a semi-empirical wake decay constant k

$$u^* = u_0\left[1 - 2a/(1 + kx/r_1)^2\right] \quad (1)$$

where u^* is the wake velocity at the downstream position x, u_0 is the incoming wind speed, a is the axial induction factor calculated from the thrust coefficient C_T of the wind turbine using the following formula

$$a = \left(1 - \sqrt{1 - C_T}\right)/2 \quad (2)$$

and r_1 is the characteristic downstream rotor radius represents the expanded wake radius immediately downstream of the wind turbine, which can be computed using

$$r_1 = r_d\sqrt{(1-a)/(1-2a)} \quad (3)$$

where r_d is the rotor radius of the wind turbine.

The Jensen model assumes that the wake expands linearly with a parameter wake decay constant k denoting the growth of the wake width per unit length in the downwind direction. The expressions for k and the spread wake radius r_x (the wake radius denotes the distance between the rotor axis and the first point where the wind speed is equal to the free stream value) are given as follows

$$k = 0.5/\ln(z/z_0); \quad r_x = kx + r_d \quad (4)$$

where z is the hub height of the wind turbine, z_0 is the surface roughness height of a local terrain.

When considering the wake flow area and the outward free stream area at a certain downstream position, it is found that the shape of the wake velocity along the cross-stream direction is like a top-hat, therefore the distribution of predicted wake velocity by the 1D Jensen model is also called a top-hat distribution. It should be mentioned that the Jensen wake model assumes the wake is fully turbulent and the contribution from the tip vortices is negligible, which leads to the fact that the Jensen model is not designed for use in the near-wake condition. It should be applicable to capture the velocity deficit for a downstream distance in excess of three rotor diameters (3D) (Barthelmie et al., 2006).

3. 2D Jensen wake models

In this section, the new 2D wake model is presented. The wake velocity distribution model is first introduced in Section 3.1. Then the wake turbulence distribution model is developed in Section 3.2 to take the turbulence into account in the wake model. In the last Section 3.3, the 2D wake model with turbulence is proposed based on the 2D Jensen model described in Section 3.1 and the wake decay parameter in the function of the wake turbulence developed in Section 3.2. Note that for the sake of simplicity, this model is called 2D_k wake model in this work.

3.1. The proposal of a 2D Jensen wake model

By letting the wake deficit only vary with the downstream distance x, the Jensen wake model is referred to as a one-dimensional model. However, according to the classical theories of shear flows in the wakes of bluff bodies (Dufresne and Wosnik, 2013) as well as the wind tunnel investigations for wake behind a single wind turbine (Chamorro and Porte-Agel, 2009), it is found that the velocity in the wake has an approximately Gaussian axisymmetric shape after a certain downstream distance. To carry out more detailed investigation of wind turbine wake, the standard Jensen model seems too simple because it only describes the center line wake deficit in the cross-stream section. Thus a proper modification is highly needed to represent the physical wake distribution.

In most cases, turbine spacing at operational offshore wind farm is currently in the range of 4D–12D (Barthelmie et al., 2010). So a good engineering wake model should have the ability to accurately predict the wake development in this range. In the work of (RÉTHORÉ, 2006), it is pointed out that the Gaussian distribution which based on the self-similar theory showed a quite large error at the near wake region, and it seems to be acceptable after six rotor diameters. But in the study by N.O. Jensen (Jensen, 1983), the author validated a cosine-shape profile of the velocity deficit in the cross-wind direction against the measurements from the Nibe wake project. Agreement between the numerical prediction and the measurements was seen to be satisfactory at both the near wake and far wake areas. Besides, in the reference (Taylor, 1990) it is mentioned that at the far wake region the velocity deficit is supposed to have adopted a self-preserving form, which is usually represented either by a Gaussian or a closely similar polynomial relation. According to these, we try to use the similar sinusoidal function to fit the profile of the wake deficit. So the simple cosine function is employed here instead of a Gaussian distribution, and hence a 2D wake velocity model is developed. The equation of the new model is defined as

$$u = A\cos(K \times r + \pi) + B \quad (5)$$

of which, A, K and B are determined constants, r is the radial distance from the center of the wake. For the sake of simplicity, this 2D wake model is called 2D Jensen model because it is based on the original Jensen wake model and the following assumptions:

- Assumption 1: The new model has the same wake radius as the original model

$$\frac{2\pi}{K} = 2r_x \quad (6)$$

- Assumption 2: When the variable r tends to the outer boundary of the wake region r_x, the mean wind speed reaches the free-stream wind speed

$$A\cos(K \times r_x + \pi) + B = u_0 \quad (7)$$

- Assumption 3: Integrating the wind speed along the cross wind direction provides the flow mass flux. In this work, the mass flux calculated by the 2D wake model is considered to be equivalent to the mass flux estimated by the Jensen model

$$\int_{-r_x}^{r_x} [A\cos(K \times r + \pi) + B]dr = u^* \times 2r_x \quad (8)$$

where u^* is the velocity predicted by the original Jensen wake